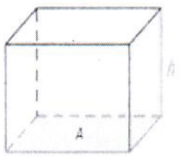
Secondary Math 2 **12.2 Notes** **Volume of Solids**

Objective: To be able to give an informal argument for the formulas of the volumes of 3-Dimensional solids.

Volumes that connect two bases

Prism

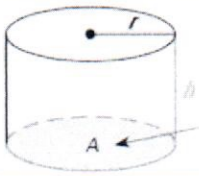
$$Volume = A_{base} \cdot h_{solid}$$



$V_{RP} = bhw$

Cylinders

$$Volume = A_{circle} \cdot h_{solid}$$

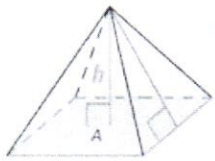


$V_{cyl} = \pi r^2 h$

Volumes that connect a base to a point

Pyramid

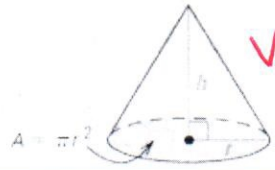
$$Volume = \frac{1}{3} A_{base} \cdot h_{solid}$$



$V_{pyr} = \frac{1}{3}bhw$

Cone

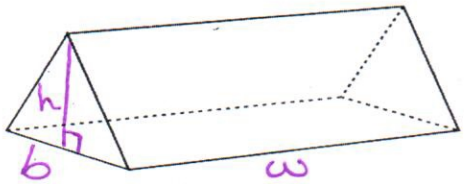
$$Volume = \frac{1}{3} A_{base} \cdot h_{solid}$$



$V_{cone} = \frac{1}{3}\pi r^2 h$

Volume of a Triangular Prism

If the volume of a solid is the area of its base multiplied by its height, what is the formula for volume of a triangular prism?



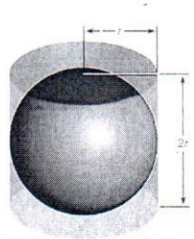
Formula:

$V_{TP} = \frac{1}{2}bhw$

Volume of a Sphere

Let's come up with the formula of the sphere:

Volume of a cylinder:	$V_{cyl} = \pi r^2 h$
Height of cylinder: (In terms of radius)	$h = 2r$
Make the necessary substitutions:	volume of a sphere is 2/3 of a cylinder
$Volume =$	$\frac{2}{3}(\pi r^2 h) = \frac{2}{3}(\pi r^2 \cdot 2r)$

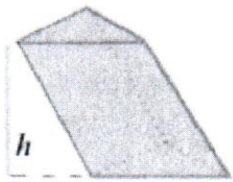


Formula:

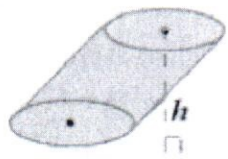
$V_S = \frac{4}{3}\pi r^3$

What about Oblique Shapes?

Cavalieri's Principle: If two solids have the same height and the same area at every level, then they will have the same volume



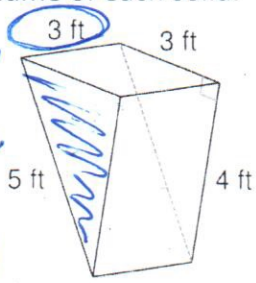
Oblique Prism



Oblique Cylinder

Guided Practice: Find the volume of each solid.

1) Triangular Prism

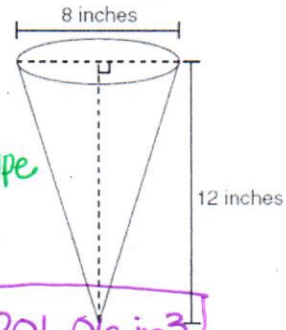


$V_{TP} = A_{base} \cdot h_{shape}$

$= \frac{1}{2}(3)(4)(3)$

$= \boxed{18\ ft^3}$

2) circular pyramid (cone)



$V_{cone} = \frac{1}{3} A_{base} \cdot h_{shape}$

$= \frac{1}{3}\pi(4)^2(12)$

$= \boxed{64\pi\ in^3}$ $\boxed{201.06\ in^3}$

3) A soup can has a radius of 4.3 cm and a height of 11.6 cm. What is the volume of the soup can to the nearest tenth of a cubic centimeter? cylinder

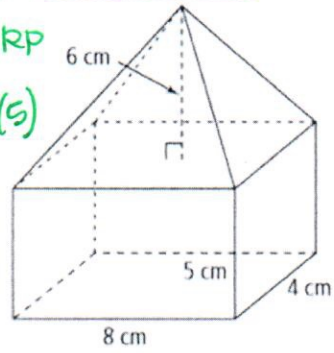
$V_{cyl} = A_{base} \cdot h_{shape}$

$= \pi r^2 h$

$= \pi(4.3)^2(11.6)$

$= \boxed{673.8\ cm^3}$

4)



$V_{shape} = V_{pyr} + V_{RP}$

$= \frac{1}{3}(8)(4)(6) + (8)(4)(5)$

$= 64 + 160$

$= \boxed{224\ cm^3}$

More Examples:

1) The volume of a cylinder is $282.74 cm^3$ and the height of the cylinder is $10 cm$. What is the radius?

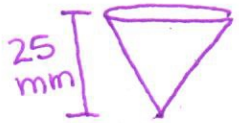
$V = 282.74\ cm^3$ r = ?

$V_{cyl} = \pi r^2 h$

$\frac{\pi r^2(10)}{10\pi} = \frac{282.74}{10\pi}$

$\sqrt{r^2} = \sqrt{9.0}$ $\boxed{r = 3.0\ cm}$

2) Volume of a cone is $10471.98 mm^3$ with height of $25 mm$. Find the diameter.



$V = 10471.98\ mm^3$ d = ? $d = 2r$

$\frac{1}{3}\pi r^2 h = V$

$3 \cdot \frac{1}{3}\pi r^2(25) = 10471.98 \cdot 3$

$\frac{\pi r^2(25)}{25} = \frac{31415.94}{25}$

$\frac{\pi r^2}{\pi} = \frac{1256.64}{\pi}$

$r^2 = 400.0$

$r = 20mm$

$\boxed{d = 40mm}$

3) If the volume of a sphere is $2304\pi ft^3$, what is the radius of the sphere?

$V = 2304\pi\ ft^3$ r = ?

$V = \frac{4}{3}\pi r^3$

$\frac{2304\pi}{\pi} = \frac{\frac{4}{3}\pi r^3}{\pi}$

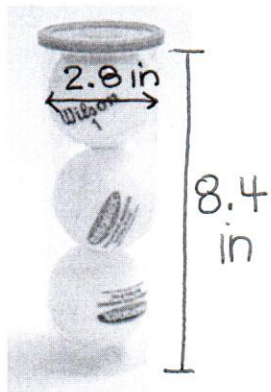
$\left(\frac{3}{4}\right)2304 = \frac{4}{3}r^3\left(\frac{3}{4}\right)$

$\sqrt[3]{r^3} = \sqrt[3]{1728}$

$\boxed{r = 12\ ft}$

Tennis Balls Task:

Tennis balls are packaged in a cylindrical container and contain 3 tennis balls (see picture). If the tennis ball has a diameter of 2.8 inches and the total height of the canister is 8.4 inches, about how much wasted space (air) is there in the container? (Assume that the tennis balls touch the side of the container and the top and bottom with no gaps.)



r = 1.4 in

air = cylinder − 3 sphere's

$= \pi(1.4)^2(8.4) - 3\left(\frac{4}{3}\pi(1.4)^3\right)$

$= 51.723 - 34.482$

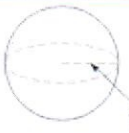
$= \boxed{17.241\ in^3}$

Secondary Math 2 12.2 In Class Assignment Name: _____ Period: _____

Volume of Solids

Find the volume of each figure. Round the nearest hundredth if necessary.

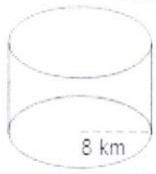
1. even



$V = \frac{4}{3}\pi(11)^3$

$V = 5575.28\ ft^3$

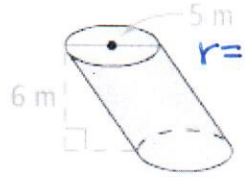
2. odd



$V = \pi(8)^2(10)$

$= 2010.62\ km^3$

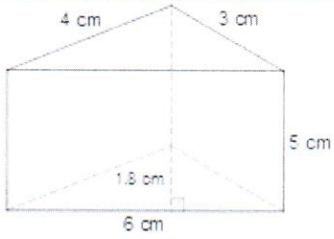
3.



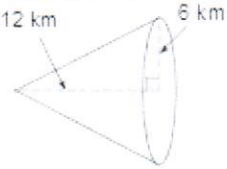
$V_{cyl} = \pi(2.5)^2(6)$

$= 117.81\ m^3$

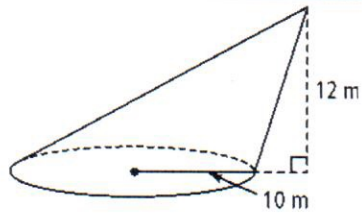
4.



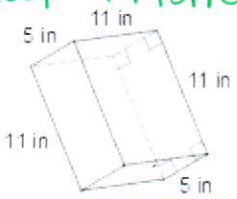
5.



6.



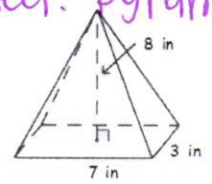
7. Rect. Prism



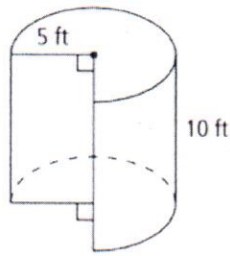
$V = (11)(11)(5)$

$= 605\ in^3$

8. Rect. Pyramid



9.

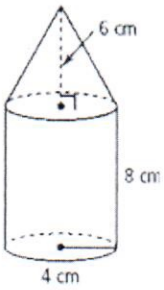


$V = \frac{3}{4}(V_{cyl})$

$= \frac{3}{4}(\pi(5)^2(10))$

$589.05\ ft^3$

10. *Note that 4cm is the diameter



11.

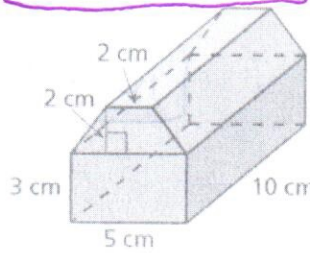
$V = \frac{1}{2}$ sphere + cylinder

$= \frac{1}{2}\left(\frac{4}{3}\pi(6)^3\right) + (\pi(6)^2(13))$

$= 452.39 + 1470.27$

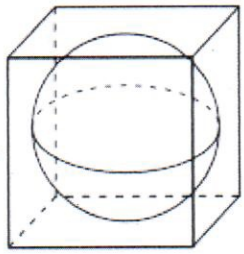
$= 1922.66\ in^3$

12.



Answer each question

13. The sphere below fits snugly inside a cube with 18 cm edges.



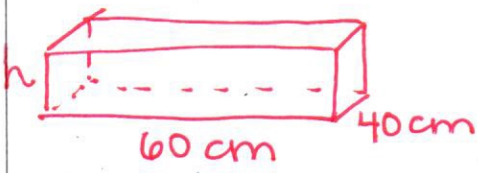
a) What is the volume of the sphere?

b) How much more volume is in the cube than in the sphere?

14. A spherical scoop of ice cream with a diameter of 4 cm rests on top of a sugar cone that is 10 cm deep and has a diameter of 4 cm. If all of the ice cream melts into the cone, what percent of the cone will be filled?

15. A recycling bin is a rectangular prism and holds 360,000 cm^3 of returnable containers. If the base of the bin is 40 cm wide and 60 cm long, what is the height of the recycling bin in meters?

$V = 360{,}000\ cm^3$



$V = bwh$

$360{,}000 = (60)(40)(h)$

$\frac{360{,}000}{2400} = \frac{2400h}{2400}$

$h = 150\ cm$

16. A juice container shaped like a cylinder has a base area of 90 cm^2 and can hold 990 cm^3 of juice. What is the height of the juice container?