

Objectives:

- I can add, subtract, and multiply complex numbers

What happens when you type $\sqrt{-1}$ in your calculator? What does this mean?

domain error / non real answer

Historical Note: French mathematician René Descartes suggested the imaginary unit i be defined so that $i^2 = -1$. The imaginary unit enables us to solve problems that we would not otherwise be able to solve. Problems involving electricity often use the imaginary unit.

Standard Form of a Complex Number:

- Form: $a + bi$
- a and b are real numbers and i is the imaginary unit.
- a is the real part of the complex number, and bi is the imaginary part of the complex number.

$i = \sqrt{-1}$

$i^2 = -1$

$i \cdot i = (\sqrt{-1})(\sqrt{-1})$

$i^3 = -i$

$i \cdot i^2 = i(-1) = -i$

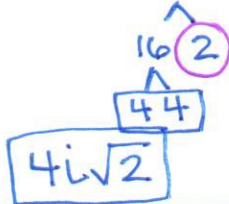
$i^4 = 1$

$i^2 \cdot i^2 = (-1)(-1) = 1$

Example 1:

Simplify $\sqrt{-32}$ by using the imaginary number i .

$\sqrt{-1 \cdot 32} = \sqrt{-1} \cdot \sqrt{32} = i\sqrt{32}$



Example 2:

Write the complex number $\sqrt{-4} + 3$ in standard form: $a + bi$

$i\sqrt{4}$
 $3 + 2i$

Example 3: Perform the indicated operation.

$(17 - 2i) + (4 + 5i)$
 $17 - 2i + 4 + 5i$
 $17 + 4 - 2i + 5i$
 $21 + 3i$

$(3 + 2i) - (-8 + 2i)$
 $3 + 2i + 8 - 2i$
 $11 + 0i$
 11

$(12 + 5i) - (2 - i)$
 $12 + 5i - 2 + i$
 $10 + 6i$

Example 4: Perform the indicated operation.

$4i(2 - 3i)$
 $4i(2) - 4i(3i)$
 $8i - 12i^2$
 $8i - 12(-1)$
 $+12$
 $12 + 8i$

$(4 - 3i)(5 + 2i)$
 $20 + 8i - 15i - 6i^2$
 $20 + -7i - 6(-1)$
 $20 - 7i + 6$
 $26 - 7i$

$\sqrt{-6} \cdot \sqrt{-2}$
 $i\sqrt{6} \cdot i\sqrt{2}$
 $i^2 \sqrt{6 \cdot 2}$
 $i^2 \sqrt{12}$
 $(-1)\sqrt{12}$
 $-\sqrt{12} < \frac{b}{2} \leq \frac{3}{2}$
 $-2\sqrt{3}$

Examples of complex numbers:

Adding Complex Numbers

$$\begin{aligned} & \text{Like Terms} \quad (8+7i) + (4+3i) \quad \text{Like Terms} \\ & = (8 + 4) + (7i + 3i) \\ & = \boxed{12 + 10i} \end{aligned}$$

$i = \sqrt{-1}$
 $7x + 3x = 10x$

Subtracting Complex Numbers

$$\begin{aligned} & \text{Like Terms} \quad (7+9i) - (3+4i) \quad \text{Like Terms} \\ & = 7 + 9i - 3 - 4i \\ & = 7 - 3 + 9i - 4i \\ & = \boxed{4 + 5i} \end{aligned}$$

$i = \sqrt{-1}$
 $9x - 4x = 5x$

MULTIPLY

$$\begin{aligned} & \boxed{(3+2i) \cdot (4+5i)} \quad \begin{array}{l} i = \sqrt{-1} \\ i^2 = -1 \end{array} \\ & = 12 + 15i + 8i + 10i^2 \\ & = 12 + 15i + 8i - 10 \\ & = \boxed{2 + 23i} \end{aligned}$$

Assignment

Date _____ Period _____

Simplify.

1) $(6 + 8i) - (-4 + 5i)$

$$10 + 3i$$

3) $(4 + 5i) + (-1 - 5i)$

$$3 \text{ OR } 3 + 0i$$

5) $(-7i) - 3 + (5 + 4i)$

$$2 - 3i$$

7) $(3 + i)(4 + 8i)$

$$12 + 24i + 4i + 8i^2$$

$$4 + 28i$$

9) $(8 - 3i)(3 + 6i)$

$$24 + 48i - 9i - 18i^2$$

$$42 + 39i$$

11) $4(2i)(-8 + 6i)$

$$(8i)(-8 + 6i)$$

$$-64i + 48i^2 = -48 - 64i$$

13) $(4 - 5i)(-6 + 5i)$

$$-24 + 20i + 30i - 25i^2$$

$$1 + 50i$$

15) $(-6 + 4i)^2 = (-6 + 4i)(-6 + 4i)$

$$36 - 24i - 24i + 16i^2$$

$$20 - 48i$$

17) $(-5 - 2i)^2 = (-5 - 2i)(-5 - 2i)$

$$25 + 10i + 10i + 4i^2$$

$$21 + 20i$$

19) $\sqrt{32x^3}$

$$\begin{array}{c} 4 \\ \swarrow \downarrow \searrow \\ 8 \\ \swarrow \downarrow \searrow \\ 4 \end{array} \begin{array}{c} x \\ x \\ x \end{array}$$

$$4x\sqrt{2x}$$

21) $\sqrt{448r^4}$

$$\begin{array}{c} \wedge \\ 28 \quad 16 \\ \wedge \quad \wedge \\ 7 \quad 4 \quad 4 \quad 4 \end{array} \begin{array}{c} r \\ r \\ r \\ r \end{array}$$

$$7 \quad 4 \quad 4 \quad 4$$

$$\begin{array}{c} \wedge \quad \wedge \quad \wedge \\ 2 \quad 2 \quad 2 \end{array}$$

$$8r^2\sqrt{7}$$

2) $(3 - 7i) + (-4 + 8i)$

4) $(3 - 5i) + 6 - (8i)$

6) $(8 + 4i) + (7 + 6i)$

8) $(6 - 5i)^2$

10) $(-1 - 8i)(5 + 6i)$

12) $(1 + i)(-6 - 7i)$

14) $(5 - 6i)(8 + 8i)$

16) $(4 + 6i)(6 - 6i)$

18) $\sqrt{256m^2}$

20) $\sqrt[3]{24x^5}$

22) $\sqrt[3]{16a^3}$