Objectives:

- This unit we have learned how to solve quadratic equations by factoring and by taking the square root. Today you will learn how to solve quadratic equations by completing the square.

Warm-Up: Multiply each binomial

1. $(x+3)^2$ 2. $(x+4)^2$ 3. $(x-5)^2$

$=(x+3)(x+3)$

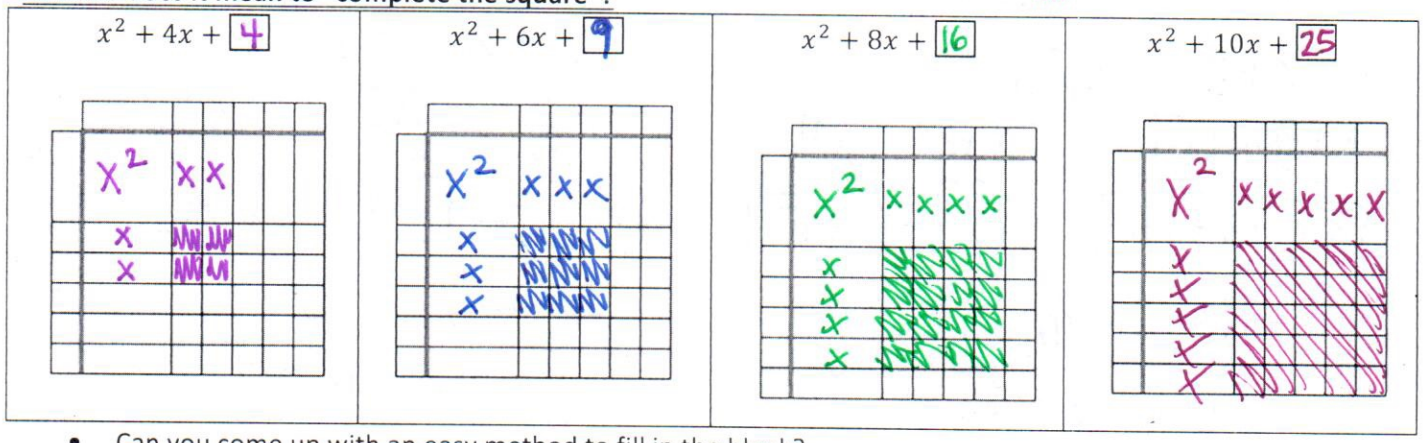
$=x^2+3x+3x+9$

x^2+6x+9 $x^2+8x+16$ $x^2-10x+25$

- What patterns do you notice?
  - all are squared
  - 3 term answers
  - std form
  - positive constant
  - combine middle terms
- When we "complete the square" we manipulate an equation so that it becomes a perfect square trinomial.

$ax^2-2ab+b^2$
$ax^2+2ab+b^2$

So what does it mean to "complete the square"?



ax^2+bx+c

- Can you come up with an easy method to fill in the blank?

$\left(\frac{b}{2}\right)^2$

Example 1: Solve $x^2-4x+2=0$

You will notice that this problem is not factorable. Because all problems are not factorable we need another method. That is where completing the square comes in.

$x^2-4x+2=0$
$\quad -2 \quad -2$

$x^2-4x=-2$

$x^2-4x+\left(\frac{b}{2}\right)^2=-2+\left(\frac{b}{2}\right)^2$

$x^2-4x+\left(\frac{4}{2}\right)^2=-2+\left(\frac{4}{2}\right)^2$

$x^2-4x+4=-2+4$

$(x-2)(x-2)=2$

$(x-2)^2=2$

$\sqrt{(x-2)^2}=\sqrt{2}$

$x-2=\pm\sqrt{2}$

$x+2=+\sqrt{2}$ $\quad$ $x+2=-\sqrt{2}$
$+2 \quad +2$ $\quad$ $+2 \quad +2$

$x=2+\sqrt{2}$ $\quad$ $x=2-\sqrt{2}$

$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$ OR $x^2 - bx + \left(\frac{b}{2}\right)^2 = \left(x - \frac{b}{2}\right)^2$

Example 2: Solve $2x^2 + 4x - 18 = 0$ by completing the square

$+18 \quad +18$

$\frac{2x^2}{2} + \frac{4x}{2} = \frac{18}{2}$

$x^2 + 2x = 9$

$x^2 + 2x + \left(\frac{2}{2}\right)^2 = 9 + \left(\frac{2}{2}\right)^2$

$x^2 + 2x + 1 = 9 + 1$

$\sqrt{(x+1)^2} = \sqrt{10}$

$x + 1 = \pm\sqrt{10}$

$x + 1 = +\sqrt{10}$ $\quad -1 \quad -1$

$x = -1 + \sqrt{10}$

$x + 1 = -\sqrt{10}$ $\quad -1 \quad -1$

$x = -1 - \sqrt{10}$

Practice Problems: Solve the quadratic equation by completing the square

1. $x^2 + 6x - 2 = 0$

$+2 \quad +2$

$x^2 + 6x = 2$

$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 2 + \left(\frac{6}{2}\right)^2$

$\sqrt{(x+3)^2} = \sqrt{11}$

$x + 3 = \pm\sqrt{11}$

$x + 3 = +\sqrt{11}$ $\quad -3 \quad -3$

$x = -3 + \sqrt{11}$

$x + 3 = -\sqrt{11}$ $\quad -3 \quad -3$

$x = -3 - \sqrt{11}$

2. $4x^2 + 16x = 36$

$\frac{4x^2}{4} + \frac{16x}{4} = \frac{36}{4}$

$x^2 + 4x = 9$

$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 9 + \left(\frac{4}{2}\right)^2$

$\sqrt{(x+2)^2} = \sqrt{13}$

$x + 2 = \pm\sqrt{13}$

$x + 2 = +\sqrt{13}$ $\quad -2 \quad -2$

$x = -2 + \sqrt{13}$

$x + 2 = -\sqrt{13}$ $\quad -2 \quad -2$

$x = -2 - \sqrt{13}$

3. $10x^2 + 84x = 4x^2 - 300$

$-4x^2 \quad -4x^2$

$\frac{6x^2}{6} + \frac{84x}{6} = \frac{-300}{6}$

$x^2 + 14x = -50$

$x^2 + 14x + \left(\frac{14}{2}\right)^2 = -50 + \left(\frac{14}{2}\right)^2$

$\sqrt{(x+7)^2} = \sqrt{-1}$

$x + 7 = \pm\sqrt{-1}$

$x + 7 = \pm i$

$x + 7 = +i$ $\quad -7 \quad -7$

$x = -7 + i$

$x + 7 = -i$ $\quad -7 \quad -7$

$x = -7 - i$

4. $3x^2 - 10x + 76 = 2x^2 + 6x$

$-2x^2 - 6x \quad -2x^2 - 6x$

$x^2 - 16x + 76 = 0$

$-76 \quad -76$

$x^2 - 16x = -76$

$x^2 - 16x + \left(\frac{16}{2}\right)^2 = -76 + \left(\frac{16}{2}\right)^2$

$\sqrt{(x-8)^2} = \sqrt{-12}$

$x - 8 = \pm\sqrt{-12}$

$x - 8 = \pm i\sqrt{12}$

$x - 8 = +i\sqrt{12}$ $\quad +8 \quad +8$

$x = 8 + i\sqrt{12}$

$x - 8 = -i\sqrt{12}$ $\quad +8 \quad +8$

$x = 8 - i\sqrt{12}$