

Objective:

- During the next two units we will be focusing on algebraic and geometric proofs. By the end of class you will be able to justify your reasoning using mathematical properties.

A true statement that follows as a result of other true statements is called a Theorem. All theorems must be proven. You can prove a theorem using a 2 column proof.

A two-column proof has numbered statements and reasons that show the logical order of an argument

Algebraic Properties of Equality

Addition Property:	If $a = b$ then $a + c = b + c$
Subtraction Property:	If $a = b$ then $a - c = b - c$
Multiplication Property:	If $a = b$ then $a \cdot c = b \cdot c$
Division Property:	If $a = b$ then $\frac{a}{c} = \frac{b}{c}, c \neq 0$
Distributive Property:	$a(b + c) = a \cdot b + a \cdot c$
Combining Like Terms:	$ax + bx + cy = (a + b)x + cy$
Substitution Property:	If $y = a$ then $y^2 = a^2$



Geometric Properties of Equality

Midpoint Theorem:	If B is the midpoint of \overline{AC} , then $\overline{AB} + \overline{BC} = \overline{AC}$
Segment Addition Postulate:	If B is co-linear to points A, C and B is between A, C then $\overline{AB} + \overline{BC} = \overline{AC}$



Example Set: Solve and fill in the reason that justifies each step.

Given: $2x - 5 = 13$
 Prove: $x = 9$


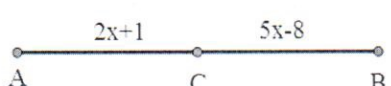

Given: C is the midpoint of \overline{AB}
 Prove: $x = 6$



Statement	Reason
1) $2x + 5 = 13$	1) Given
2) $\frac{2x}{2} = \frac{18}{2}$	2) Addition property
3) $x = 9$	3) Division property

Statement	Reason
1) C is the midpoint of \overline{AB}	1) given
2) $\overline{AC} \cong \overline{CB}$	2) C is midpoint
3) $5x - 16 = 2x + 2$	3) substitution
4) $3x - 16 = 2$	4) combine like terms
5) $3x = 18$	5) Addition property
6) $x = 6$	6) division property

Practice Problems: Solve and fill in the reason that justifies each step.

<p>1) Given: $5x - 18 = 3x + 2$ Prove: $x = 10$</p> <table border="1"> <thead> <tr> <th>Statement</th> <th>Reason</th> </tr> </thead> <tbody> <tr> <td>1) $5x - 18 = 3x + 2$</td> <td>1) Given</td> </tr> <tr> <td>2) $5x = 3x + 20$</td> <td>2) Addition prop.</td> </tr> <tr> <td>3) $2x = 20$</td> <td>3) subtraction prop.</td> </tr> <tr> <td>4) $x = 10$</td> <td>4) Division Prop.</td> </tr> </tbody> </table>	Statement	Reason	1) $5x - 18 = 3x + 2$	1) Given	2) $5x = 3x + 20$	2) Addition prop.	3) $2x = 20$	3) subtraction prop.	4) $x = 10$	4) Division Prop.	<p>2) Given: $\frac{1}{2}x + 12 = -18$ Prove: $x = -60$</p> <table border="1"> <thead> <tr> <th>Statement</th> <th>Reason</th> </tr> </thead> <tbody> <tr> <td>1) $\frac{1}{2}x + 12 = -18$</td> <td>1) given</td> </tr> <tr> <td>2) $\frac{1}{2}x = -30$</td> <td>2) subtraction prop.</td> </tr> <tr> <td>3) $x = -60$</td> <td>3) Multiplication prop.</td> </tr> </tbody> </table>	Statement	Reason	1) $\frac{1}{2}x + 12 = -18$	1) given	2) $\frac{1}{2}x = -30$	2) subtraction prop.	3) $x = -60$	3) Multiplication prop.										
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