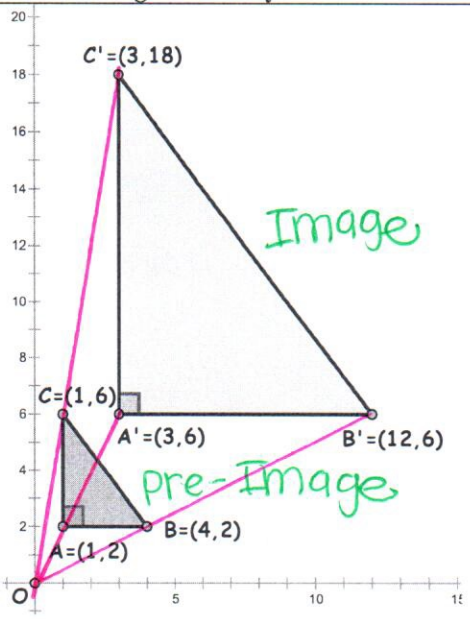
Discovering Similarity:



Vocabulary:

1. **Dilation**: is a transformation that produces an image that is the same shape as the original figure but the image is a different size. The dilation uses a center and a scale factor to create a proportional figure.
2. **Center of Dilation**: is a fixed point in the plane about which all points are expanded or contracted.
3. **Scale Factor**: is the ratio of the new image to the original image (i.e. if the original figure has a length of 2 and the new figure has a length of 4, the scale factor is $\frac{4}{2} = 2$.
4. **Pre-image**: the original figure before the transformation takes place. A
5. **Image**: the figure after the transformation has taken place. *(Prime)* A'
6. **Types of Dilations**: Reduction, Congruency, Enlargement

Use the figure above to answer questions 1-7

1. Calculate the lengths of the following segments:
   In case you forget, the distance formula is:
   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

   $\overline{A'B'} = 9$ $= \sqrt{(12-3)^2 + (6-6)^2} = \sqrt{9^2 + 0^2} = \sqrt{81}$

   $\overline{A'C'} = 12$ $\sqrt{(3-3)^2 + (18-6)^2} = \sqrt{0^2 + 12^2} = \sqrt{144}$

   $\overline{B'C'} = 15$ $\sqrt{(3-12)^2 + (18-6)^2} = \sqrt{(-9)^2 + 12^2} = \sqrt{225}$

   $\overline{AB} = 3$ $\sqrt{(1-4)^2 + (2-2)^2}$

   $\overline{AC} = 4$ $\sqrt{(1-1)^2 + (6-2)^2}$

   $\overline{BC} = 5$ $\sqrt{(1-4)^2 + (6-2)^2}$

2. What interesting things did you notice about your answers to #1?

   $\frac{\text{Image}}{3} = \text{pre-Image}$ OR pre-Image $\boxed{\times 3} = \text{Image}$ scale factor

3. What do you notice about the angles in each of the triangles?

   angles stay the same

4. Find the slope of $\overline{BC}$ and $\overline{B'C'}$. Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

   B(4,2) C(1,6) B'(12,6) C'(3,18)

   $m_{BC} = \frac{6-2}{1-4} = \frac{4}{-3}$

   $m_{B'C'} = \frac{18-6}{3-12} = \frac{12}{-9} = \frac{4}{-3}$

5. What interesting things did you notice about your answers to #4?

   slopes are the same

6. Using a ruler, draw 3 line segments:
   a. Connect O to A'
   b. Connect O to B'
   c. Connect O to C'.

7. What interesting things did you notice about what happened on #6?

   Lines pass through all pre-image points
   • connects O to A to A'

8. If the scale factor is less than 1, how would the pre-image and image compare?

   Reduction → smaller

9. What can you conclude, using the scale factor, about an image and pre-image?

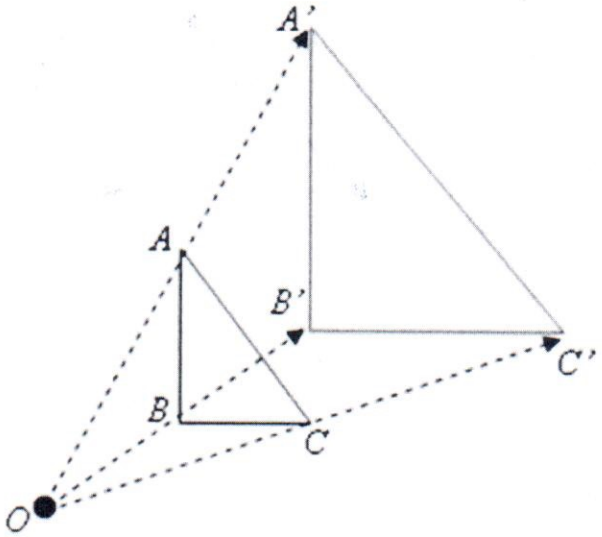
STAMP QUESTION

Objectives:

- I can recognize, perform, and interpret dilations.

Vocabulary:

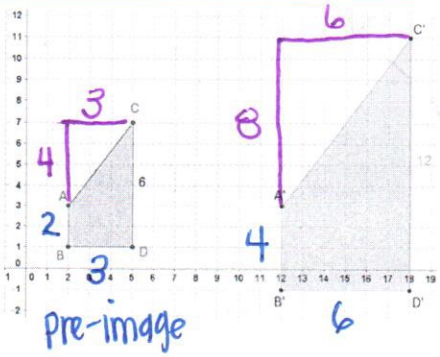
1. **Dilation**: is a transformation that produces an image that is the same shape as the original figure but the image is a different size. The dilation uses a center and a scale factor to create a proportional figure.
2. **Center of Dilation**: is a fixed point in the plane about which all points are expanded or contracted.
3. **Pre-image**: the original figure before the transformation takes place.
4. **Image**: the figure after the transformation has taken place. *(PRIME)*
5. **Scale Factor (k)**: is the ratio of the image to the pre- image (i.e. if the original figure has a length of 2 and the new figure has a length of 4, the scale factor is $\frac{4}{2} = 2$. $(k = \frac{image}{pre-image})$ Simplify
6. **Types of Dilations**: Reduction, Congruency, Enlargement



Properties of Dilation:

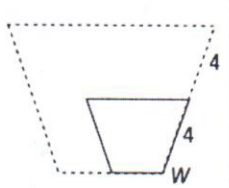
1. Angle Measure is Preserved
2. Orientation is Preserved
3. Corresponding Vertices are collinear with the center of dilation
4. Side lengths are proportional

Verify that $A'B'D'C'$ is a dilation of $ABDC$:



Example Set 1: **The following figures are dilations. Decide whether each is an enlargement or a reduction. Then find the scale factor of the dilation.** ***Solid lined figures are Pre-Images***

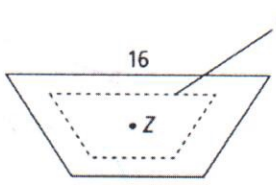
1.



enlargement

$k = \frac{8}{4} = 2$

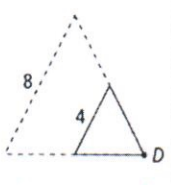
2.



Reduction

$k = \frac{9}{16}$

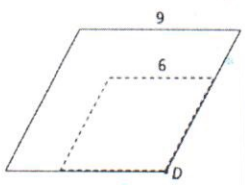
3.



enlargement

$k = \frac{8}{4} = 2$

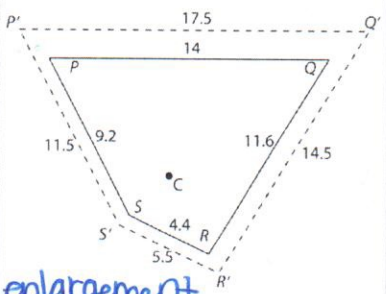
4.



Reduction

$k = \frac{6}{9} = \frac{2}{3}$

5.



enlargement

$k = \frac{17.5}{14} = \frac{5}{4}$

What patterns do you notice with the ***scale factor/k***?

When $k > 1$, the shape enlarges When $k < 1$, the shape reduces When $k = 1$, the shape stays the same

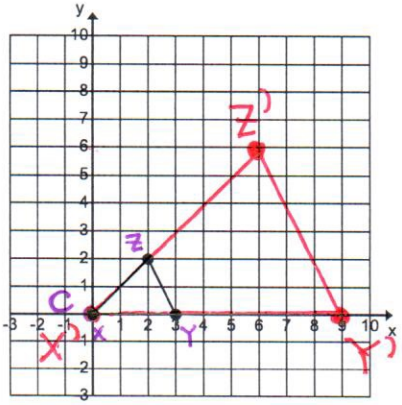
Example Set 2: Draw the dilation image of each pre-image with the given center and scale factor.

1. Center(0,0) Scale factor of 3

X ↑0 →0 ×3 ×3 ↑0 →0

Y ↑0 →3 ×3 ×3 ↑0 →9

Z ↑2 →2 ×3 ×3 ↑6 →6

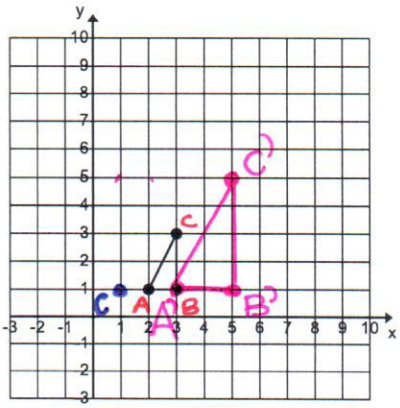


2. Center (1,1) Scale factor 2

A ↑0 →1 ×2 ×2 ↑0 →2

B ↑0 →2 ×2 ×2 ↑0 →4

C ↑2 →2 ×2 ×2 ↑4 →4



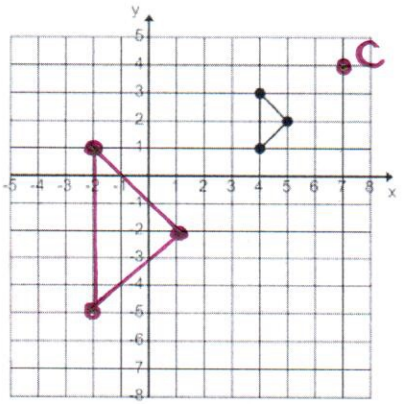
Practice Problems:

1. Center (7,4) Scale factor 3

↓1 ←3 ×3 ×3 ↓3 ←9

↓2 ←2 ×3 ×3 ↓6 ←6

↓3 ←3 ×3 ×3 ↓9 ←9



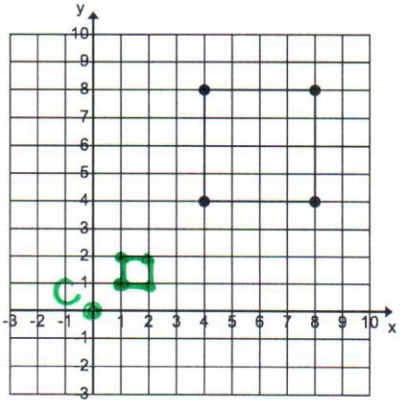
2. Center (0,0) Scale factor ¼

↑4 →4 ×1/4 ×1/4 ↑1 →1

↑8 →4 ×1/4 ×1/4 ↑2 →1

↑8 →8 ×1/4 ×1/4 ↑2 →2

↑4 →8 ×1/4 ×1/4 ↑1 →2



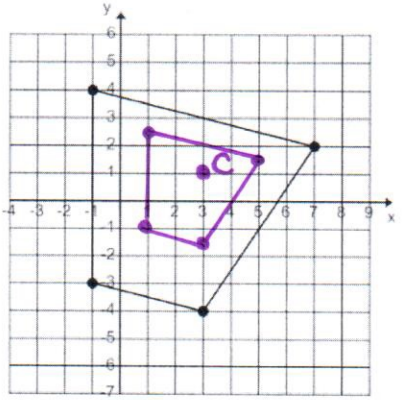
3. Center (3,1) Scale factor ½

↑1 →4 ×1/2 ×1/2 ↑1/2 →2

↑3 ←4 ×1/2 ×1/2 ↑3/2 ←2

↓5 →0 ×1/2 ×1/2 ↓5/2 →0

↓4 ←4 ×1/2 ×1/2 ↓2 ←2



4. **Graph the pre-image with given vertices.**
P(0, 4), E(6, 6), N(4, 0), T(0, -4), A(-2, 0)

Then graph the image with center of dilation at the origin and a scale factor of $\frac{1}{2}$.

P ↑4 →0 ×1/2 ×1/2 ↑2 →0

E ↑6 →6 ×1/2 ×1/2 ↑3 →3

N ↑0 →4 ×1/2 ×1/2 ↑0 →2

T ↓4 ←0 ×1/2 ×1/2 ↓2 →0

A ↑0 ←2 ×1/2 ×1/2 ↑0 ←1