

**Objectives:**

- Students will understand and can use exponent rules to simplify and evaluate basic expressions

**What do Exponents mean?** Pick the correct expression.

1.  $4 \times 4 \times 4 \times 4 \times 4 \times 4$   
 a.  $4^6$   
 b.  $6^4$   
 c.  $4^4$
2.  $3 \times 3$   
 a.  $2^3$   
 b.  $3^9$   
 c.  $3^2$
3.  $2 \times 2 \times 2 \times 2 \times 2$   
 a.  $5^2$   
 b.  $2^2$   
 c.  $2^5$

4. Explain why  $2^5$  and  $5^2$  do not give the same result.

$2 \times 2 \times 2 \times 2 \times 2 = 32$   
 $5 \times 5 = 25$

5. A student evaluated  $4^3$  to be 12. What was the student's error? What would you do to help this student understand?

$4^3 = 4 \times 3 = 12$   
 $4^3 = 4 \times 4 \times 4 \neq 12$

**Understanding exponents with zero power and negative power:**

Starting with question 6, answer questions 6-9, then fill out the table below.

x	$x^3$	$x^2$	$x^1$	$x^0$	$x^{-1}$	$x^{-2}$	$x^{-3}$
2	8 $2 \times 2 \times 2$	4 $2 \times 2$	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
3							
4							
5	225	25	5	1	$\frac{1}{5}$	$\frac{1}{25}$	$\frac{1}{125}$
10							

6. Look at the numbers going from **left to right** in the first row. Notice the pattern of dividing by two:  $8 \div 2 = 4$ ;  $4 \div 2 = 2$ . What happens if you continue this pattern going from **left to right**. What would be  $2^0$ ? What would  $2^{-1}$  be (in fraction form)? Fill in the rest of the row for "2". Then fill

7. Do negative exponents make bigger or smaller numbers?

smaller (fractions)

- ★ 8. What does  $\frac{1}{x^2}$  equal to using negative exponents?

$x^{-2}$

9. Explain why  $anything^0 = 1$

$\frac{\text{anything}}{\text{itself}} = 1$

① **The Product Rule:** Write out the following expressions in expanded form and then evaluate or simplify.

#	Expression	Expand	Simplify
10.	$2^4$	$2 \times 2 \times 2 \times 2$	16
11.	$(-2)^4$	$-2 \times -2 \times -2 \times -2$	16
12.	$-2^4$	$-(2^4) = -(2 \times 2 \times 2 \times 2)$	-16
13.	$(5^2)(5^7)$ $5^{2+7}$	$(5 \times 5) \times (5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5)$	$5^9 = 1,953,125$
14.	$ab^5 \cdot 8a^2b^5$ $8(a)(a^2)(b^5)(b^5)$ $a^3 b^{10}$	$8a b b b b b \cdot 8a a b b b b b$	$8a^3 b^{10}$

15. What is the rule for multiplying expressions with exponents?

$$(x^a)(x^b) = x^{a+b}$$

② **The Quotient Rule:** Expand the numerator and denominator of each algebraic fraction then simplify.

#	Fractions	Expand	Simplified Form
16.	$\frac{3^3}{3^2}$	$\frac{3 \times 3 \times 3}{3 \times 3}$	3
17.	$\frac{b^9}{b^6}$	$b b b b b b b b b$ $b b b b b b$ $b^{9-6}$	$b^3$
18.	$\frac{3x^6}{6x^{10}}$	$\frac{3 \times x \times x \times x \times x \times x}{3 \cdot 2 \times x \times x \times x \times x \times x \times x \times x \times x}$ $x^{6-10}$	$\frac{1}{2x^4}$ OR $\frac{1}{2}x^{-4}$ OR $\frac{x^{-4}}{2}$

19. What is the rule for dividing expressions with exponents?

$$\frac{x^a}{x^b} = x^{a-b}$$

20. Explain how it is different when the exponent on the bottom is bigger. How does that relate to negative exponents?

get negative exponents  
turn into a fraction.

③ **The Power Rule:** Write out the following expressions in expanded form and then simplify. Do not evaluate.

#	Expression	Expand	Simplify
21.	$(3^2)^4$	$(3 \times 3)^4 = (3 \times 3)(3 \times 3)(3 \times 3)(3 \times 3)$	$3^8$
22.	$(ac^2)^3$	$(ac^2)(ac^2)(ac^2)$	$a^3 c^6$
23.	$(-3c^5)^2$	$(-3c^5)(-3c^5) = (-3cccc)(-3cccc)$	$9c^{10}$
24.	$-3(c^5)^2$	$-3(c^5)(c^5) = -3(cccccc \times ccccc)$	$-3c^{10}$
25.	$\left(\frac{2x}{3y^2}\right)^3$	$\left(\frac{2x}{3y^2}\right)\left(\frac{2x}{3y^2}\right)\left(\frac{2x}{3y^2}\right) = \frac{2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x}{3 \cdot 3 \cdot 3 \cdot y^2 \cdot y^2 \cdot y^2}$	$\frac{8x^3}{27y^6}$

26. Show how you would simplify:  $\left(\frac{x^4 y^3}{z^2}\right)^5$

$$\frac{(x^4)^5 (y^3)^5}{(z^2)^5} = \frac{x^{20} y^{15}}{z^{10}}$$

27. What is the rule for raising a power to a power?

$$(a^m b^n)^p = a^{mp} b^{np}$$

Let's formulate all of these exponent rules:

<p>Product of Powers: <math>x^a \cdot x^b</math></p> $x^{a+b}$ <p><b>Numerical Example:</b></p> $x \cdot x^2 = x^{1+2} = x^3$	<p>Quotient of Powers: <math>\frac{x^a}{x^b}</math></p> $x^{a-b}$ <p><b>Numerical Example:</b></p> $\frac{3x}{x^5} = 3x^{1-5} = 3x^{-4}$ $\frac{3}{x^4} \text{ OR } \frac{3}{x^4}$	<p>Negative Exponents: <math>x^{-a}</math></p> $\frac{1}{x^a}$ <p><b>Numerical Example:</b></p> $15x^{-2} = \frac{15}{x^2}$ $2^{-1}x^3 = \frac{x^3}{2}$
<p>Power of a Power: <math>(x^a)^b</math></p> $x^{a \cdot b}$ <p><b>Numerical Example:</b></p> $(x^5)^3 = x^{5 \cdot 3} = x^{15}$ $(5x^2)^3 = 5^3(x^2)^3 = 225x^6$	<p>Zero Exponents: <math>x^0</math></p> $= 1$ <p><b>Numerical Example:</b></p> $15^0 = 1$ $(1763982)^0 = 1$	<p>Powers of a Quotient: <math>\left(\frac{x}{y}\right)^a</math></p> $\frac{x^a}{y^a}$ <p><b>Numerical Example:</b></p> $\left(\frac{p}{p^5}\right)^3 = \frac{p^3}{p^{15}} = \frac{1}{p^{12}} \text{ OR } p^{-12}$ $\left(\frac{2y}{3x^2}\right)^4 = \frac{2^4 y^4}{3^4 (x^2)^4} = \frac{2^4 y^4}{3^4 x^8}$

\*As always in Algebra, we must follow order of operations (PEMDAS).

**Guided Practice:** Simplify each expression. Use only positive exponents in your answers.

1.  $3y^8 \cdot 2y^2$

$$y^8 \cdot y^2 = y^{8+2}$$

$$6y^{10}$$

2.  $\left(\frac{c^5}{b^7}\right)^{10}$

$$\frac{(c^5)^{10}}{(b^7)^{10}} = \frac{c^{50}}{b^{70}}$$

3.  $(-2a^2w^3y)^3$

$$(-2)^3(a^2)^3(w^3)^3(y)^3$$

$$-8a^6w^9y^3$$

4.  $\left(\frac{3x^3y^{17}z}{12a^{115}b}\right)^0$

$$= 1$$

5.  $\left(\frac{1}{2}w^{-3}\right)^2 (w^4)^2$

$$\left(\frac{1}{2w^3}\right)^2 (w^4)^2$$

$$\left(\frac{1^2}{2^2(w^3)^2}\right) (w^4)^2$$

$$= \frac{1}{4w^6} (w^8)$$

$$= \frac{w^8}{4w^6}$$

$$= \frac{w^2}{4}$$

$w^{8-6} = w^2$

6.  $\left(\frac{-18x^0a^{-3}}{6(x^{-2}a^{-3})(x^{-3}a^3)}\right)^2$

$$= \left(\frac{-18x^2a^3x^3}{6a^3a^3}\right)^2$$

$$= \left(\frac{-3x^5a^3}{1a^6}\right)^2$$

$$= \left(\frac{-3x^5}{a^3}\right)^2$$

$$= \frac{(-3)^2 (x^5)^2}{(a^3)^2} = \frac{9x^{10}}{a^6}$$