Secondary 3 Honors **Name:** ______________________

Notes 12-1: Parametric Equations **Period:** ______

Watch this video as a class: https://www.khanacademy.org/math/algebra-home/alg-trig-functions/alg-parametric/v/parametric-equations-1

So far in your math classes we have represented the graph of a curve in the *xy*-plane using a single equation involving two variables, *x* and *y*. In this lesson, you will represent some of those graphs using two equations by introducing a third variable, *t*. parameter

Consider the three graphs below, each of which models different aspects of what happens when a certain object is thrown into the air.

- Figure 7.5.1 shows the vertical distance the object travels as a function of time.
- Figure 7.5.2 shows the object's horizontal distance as a function of time.
- Figure 7.5.3 shows the object's vertical distance as a function of its horizontal distance.

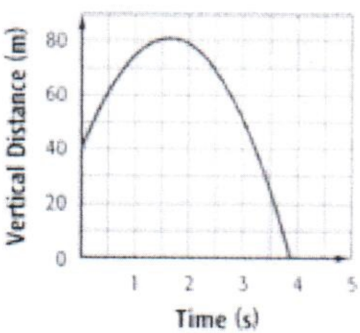


Figure 7.5.1

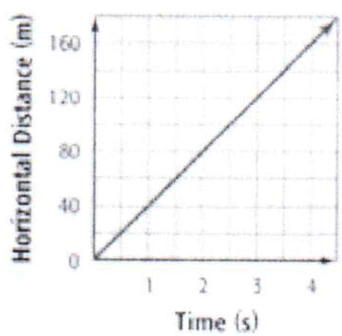


Figure 7.5.2

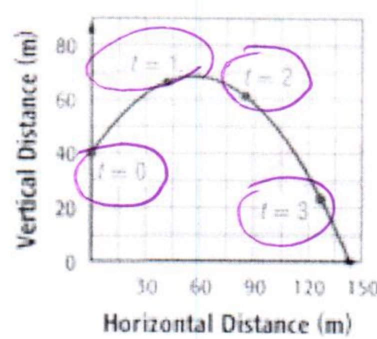


Figure 7.5.3

The first two graphs and their equations tell part of what is happening in this situation, but not the whole story. To express the position of the object, both horizontally and vertically, as a function of time, we can use **parametric equations**. The equations below both represent the graph shown in Figure 7.5.3.

Rectangular Equation

$$y = -\frac{2}{225}x^2 + x + 40$$

Parametric Equations

$$\begin{cases} x = 30\sqrt{2}t & \text{Horizontal component} \\ y = -16t^2 + 30\sqrt{2}t + 40 & \text{Vertical component} \end{cases}$$

From the parametric equations, we can now determine where the object was at a given time by evaluating the horizontal and vertical components for *t*. For example, when *t=0*, the object was at (0, 40). The variable *t* is called a parameter.

Activity #1

Standing side by side, Peyton and Eli Manning throw a football at exactly the same time. Peyton throws the ball with an initial velocity of 20 meters per second at 60°. Eli throws the ball 15 meters per second at 45°. Assuming the footballs were thrown from the same initial height, simulate the throws on a graphing calculator. The parametric equations for each throw are as follows:

Peyton Manning

$$x = 10t$$
$$y = 10\sqrt{3}t - 4.9t^2$$

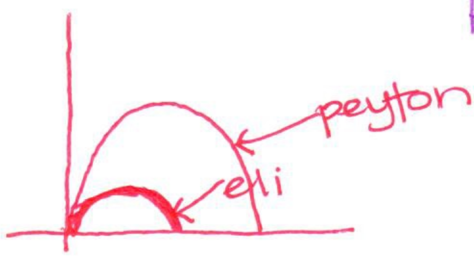
Eli Manning

$$x = 7.5\sqrt{2}t$$
$$y = 7.5\sqrt{2}t - 4.9t^2$$

Set the mode in your calculator to be degree, par, and simul. This allows the equations to be graphed simultaneously. After entering the equations, set the second set of equations to shade dark to distinguish between the throws. Set the *t*-values to range from 0 to 8, *x*-values to range from 0 to 50, and *y*-values to range from 0 to 30.

Draw the graph below. Who throws the ball the highest? Whose ball lands first?

Peyton Eli

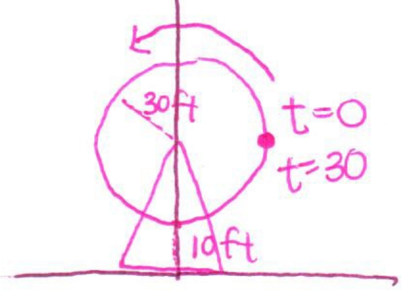


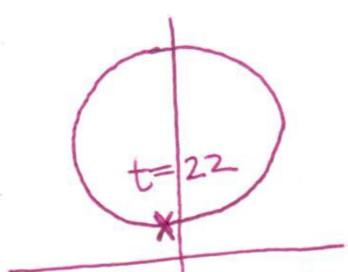
Activity #2

Theron is riding on a Ferris wheel with a radius of 30 feet. The wheel is turning counterclockwise at the rate of one revolution every 30 seconds. Assume that the lowest point of the Ferris wheel (6 o'clock) is 10 feet above the ground and Theron is at the point farthest to the right (3 o'clock) at time *t=0.*

Use the parametric equations given to find Theron's position 22 seconds into the ride. (Set calculator to RADIAN mode!)

$$x = 30\cos\left(\left(\frac{\pi}{15}\right)t\right) \qquad y = 40 + 30\sin\left(\left(\frac{\pi}{15}\right)t\right)$$





(-3.14, 10.16)

Activity #3

Two opposing players in "Capture the Flag" are 100 feet apart. On~~e~~ a signal, they run to capture a flag that is on the ground midway between them. The faster runner, however, hesitates for 0.1 sec. The following parametric equations model the race to the flag:

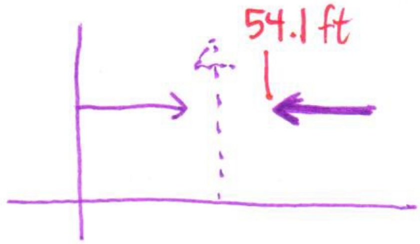
$$x_1 = 10(t - 0.1) \qquad y_1 = 3$$

$$x_2 = 100 - 9t \qquad y_2 = 3$$

Simulate the game in a [0, 100] by [-1, 10] viewing window with *t* starting at 0. Graph simultaneously. By lowering your *t*-step, you can slow down the simulation.

Who captures the flag, and by how many feet?

First Runner



$$\frac{50}{10} = \frac{\cancel{10}(t-0.1)}{\cancel{10}}$$

$$5 = t - 0.1$$
$$+0.1 \qquad +0.1$$
$$t = 5.1 \text{ sec}$$

$$X_2 = 100 - 9(5.1)$$
$$X_2 = 54.1 \text{ ft}$$
$$\boxed{4.1 \text{ ft}}$$

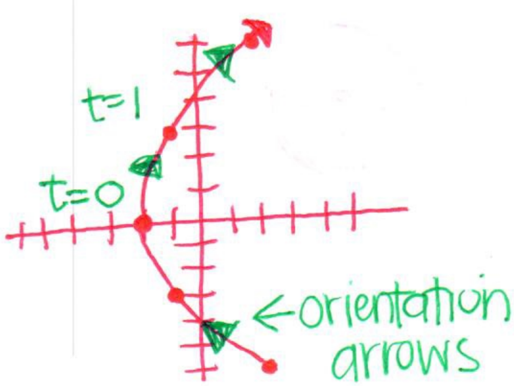
Graphing a Parametric Equation by Hand

Graph the following parametric equations without a calculator.

$x = t^2 - 2$
$y = 3t$

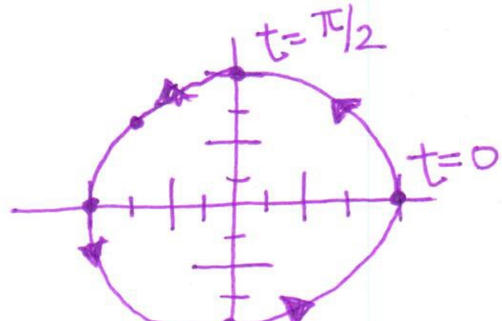
t	x	y
-2	2	-6
-1	-1	-3
0	-2	0
1	-1	3
2	2	6

(2, -6)



$x = 2\cos t$
$y = 2\sin t$

t	x	y
0	2	0
$\pi/2$	0	2
$3\pi/4$	$-\sqrt{2}/2 = -1.4$	1.41
π	-2	0
$3\pi/2$	0	-2
2π	2	0



Eliminating the Parameter

When a curve is defined parametrically it is sometimes possible to "eliminate the parameter" to obtain a rectangular equation in *x* and *y* to represents the curve. This often helps us identify the graph of a parametric curve.

Eliminate the parameter and then graph. Identify the type of parametric curve.

$x = 1 - 2t$
$y = 2 - t$

solve for t.

$x = 1 - 2t$
$\;\;-1 \;\;\; -1$

$\frac{x-1}{-2} = \frac{-2t}{-2}$

$t = -\frac{1}{2}x + \frac{1}{2}$

substitute

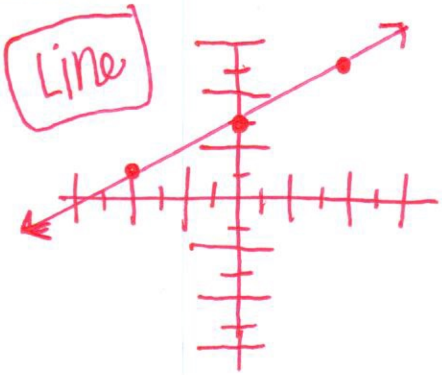
$y = 2 - \left(-\frac{1}{2}x + \frac{1}{2}\right)$

$y = 2 + \frac{1}{2}x - \frac{1}{2}$

$\boxed{y = \frac{1}{2}x + \frac{3}{2}}$

rectangular

Line



$x = t^2 - 2$
$y = 3t$

solve for t

$x = t^2 - 2$

$x + 2 = t^2$

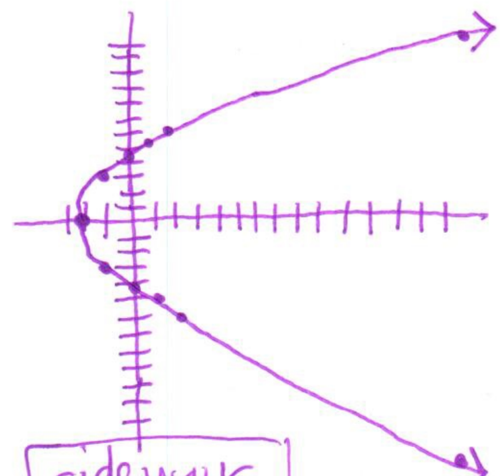
$t = \pm\sqrt{x+2}$

substitute

$\boxed{y = \pm 3\sqrt{x+2}}$

rectangular

x	y
−2	0
−1	3, −3
0	4.24, −4.24
1	5.2, −5.2
2	6, −6
14	12, −12



sideways parabola

$x = 2\cos t$
$y = 2\sin t$

$x^2 + y^2 = r^2$

$\sin^2 x + \cos^2 x = 1$

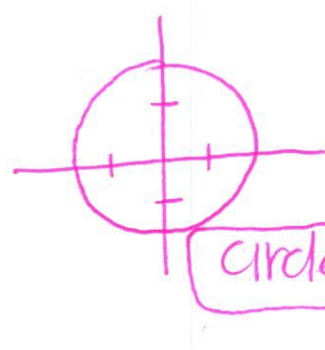
$x^2 = 4\cos^2 t$
$+\; y^2 = 4\sin^2 t$

$x^2 + y^2 = 4\cos^2 t + 4\sin^2 t$

$x^2 + y^2 = 4(\underbrace{\cos^2 t + \sin^2 t}_{=1})$

$\boxed{x^2 + y^2 = 4}$ rectangular equation

$r = 2$
center $(0,0)$



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