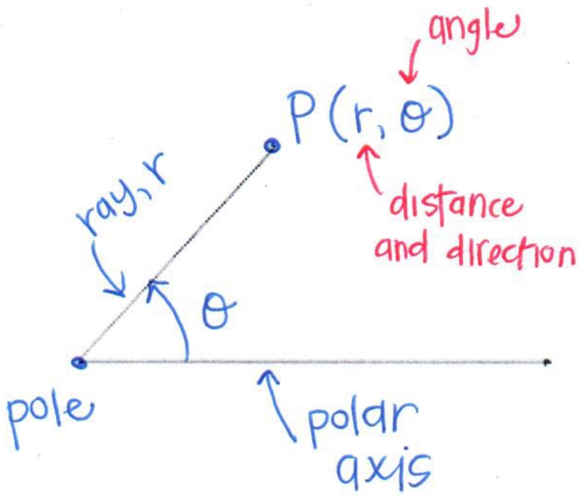
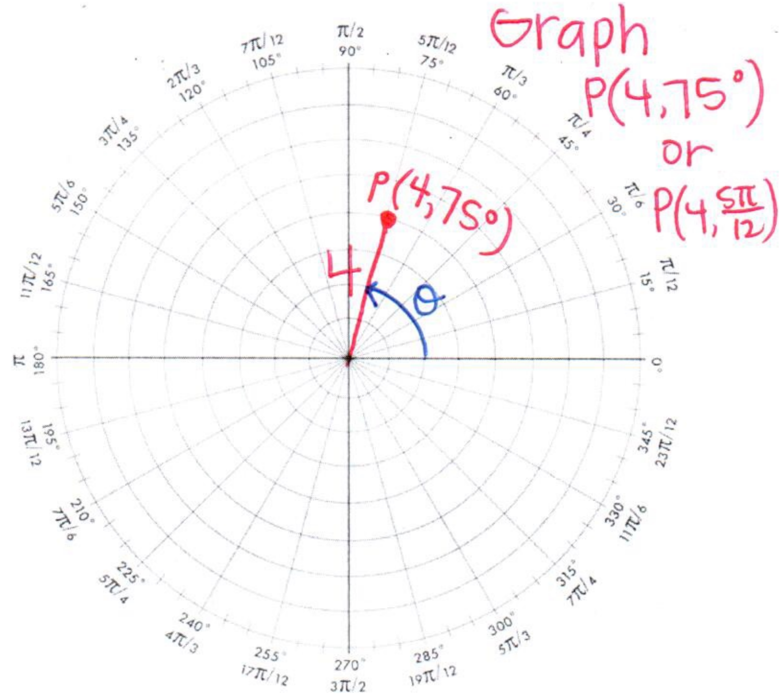
Secondary 3 Honors
Notes 12-2: Polar Coordinates

Name: answer key
Period: ______

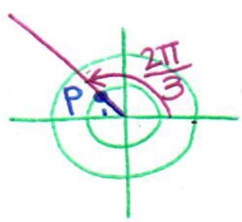
POLAR COORDINATES



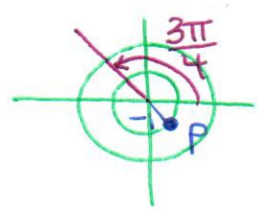


Ex. 1: Plot the points with the given polar coordinates.

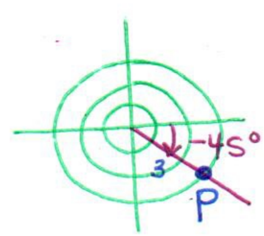
a) $P\left(1, \frac{2\pi}{3}\right)$



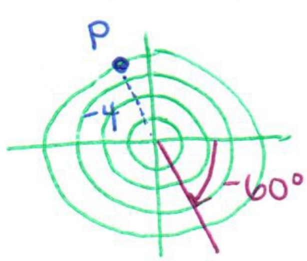
b) $P\left(-1, \frac{3\pi}{4}\right)$



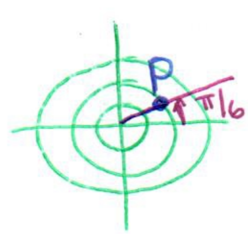
c) $P(3, -45°)$



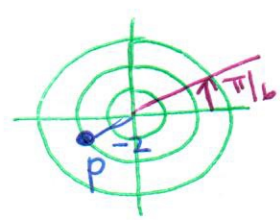
d) $P(-4, -60°)$



e) $P\left(2, \frac{\pi}{6}\right)$



f) $P\left(-2, \frac{\pi}{6}\right)$



rectangular $(x, y) \longleftrightarrow (r, \theta)$ polar

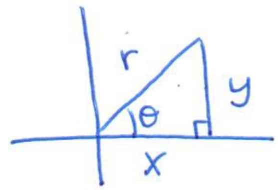
COORDINATE CONVERSION EQUATIONS

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2 \quad (a^2+b^2=c^2)$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$



$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Ex. 2: Use algebra to find the rectangular coordinates (x, y) of the points with the given polar coordinates.

a) $P\left(3, \frac{5\pi}{6}\right)$ (r, θ)

$x = r\cos\theta$

$y = r\sin\theta$

$x = 3\cos\frac{5\pi}{6} = -2.6$

$y = 3\sin\frac{5\pi}{6} = 1.5$

$P(-2.6, 1.5)$

b) $Q(2, -200°)$ (r, θ)

$x = 2\cos(-200°) = -1.9$

$y = 2\sin(-200°) = 0.7$

$Q(-1.9, 0.7)$

Ex. 3: Use algebra to find the polar coordinates (r, θ) of the points with given rectangular coordinates.

a) $(-1, 1)$

$x^2 + y^2 = r^2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$r^2 = (-1)^2 + (1)^2$

$r^2 = 2 \quad r = \pm\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-1}{1}\right)$

$\theta = -45°$ or $135°$

$(-\sqrt{2}, -45°)$

$(\sqrt{2}, 135°)$

b) $(-3, 0)$

$r^2 = (-3)^2 + 0^2$

$r^2 = 9$

$r = \pm 3$

$\theta = \tan^{-1}\left(\frac{0}{-3}\right) = \tan^{-1} 0$

$\theta = 0°$ or $180°$

$(-3, 0°)$ $(3, 180°)$

CONVERTING ENTIRE EQUATIONS

a) $r = 4\cos\theta$ polar → rectangular

$x = r\cos\theta$

$r \cdot r = r \cdot 4\cos\theta$

$r^2 = 4r\cos\theta$

$x^2 + y^2 = 4x$

b) $(x-3)^2 + (y-2)^2 = 13$ rectangular → polar

$(x-3)(x-3) + (y-2)(y-2) = 13$

$x^2 - 3x - 3x + 9 + y^2 - 2y - 2y + 4 = 13$

$x^2 - 6x + y^2 - 4y + 13 = 13$

$-13 \quad -13$

$x^2 + y^2 - 6x - 4y = 0$

($x^2+y^2 \to r^2$, $x \to r\cos\theta$, $y \to r\sin\theta$)

$r^2 - 6r\cos\theta - 4r\sin\theta = 0$

Ex. 4: Convert the equations into either polar coordinate or rectangular coordinates.

a) $r = 4\sec\theta$ polar → rectangular

$\cos\theta \cdot r = 4\left(\frac{1}{\cos\theta}\right) \cdot \cos\theta$

$r\cos\theta = 4$

$\boxed{x = 4}$

b) $x^2 + (y-1)^2 = 1$ rect → polar

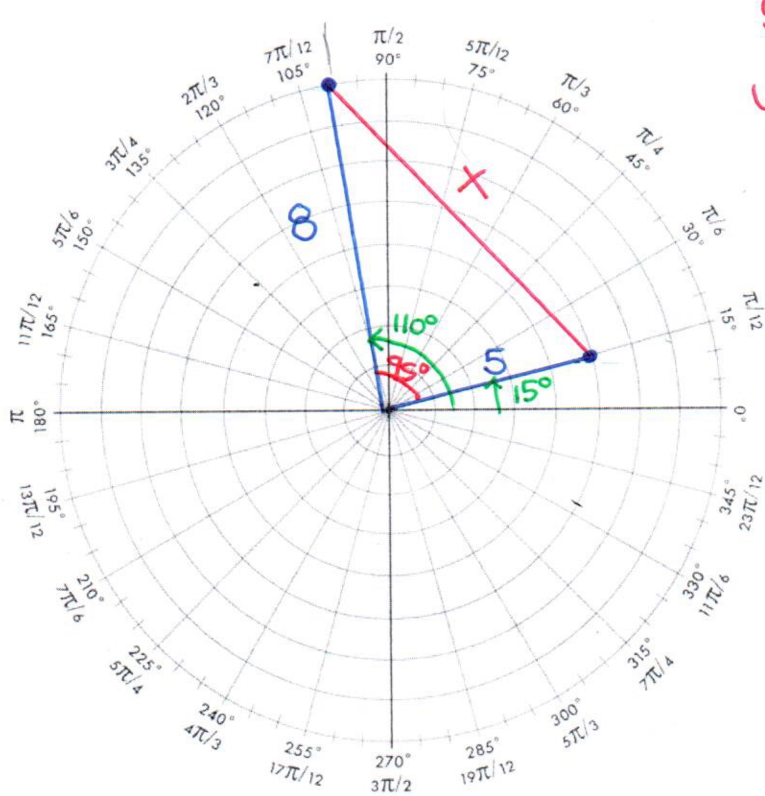
$x^2 + (y-1)(y-1) = 1$

$x^2 + y^2 - 2y + 1 = 1$
$\quad -1 \quad -1$

$x^2 + y^2 - 2y = 0$

$\boxed{r^2 - 2r\sin\theta = 0}$

Ex. 5: Using a Radar Tracking System. Radar detects two airplanes at the same altitude. Their polar coordinates are (8 mi, 100°) and (5 mi, 15°). How far apart are the planes? Start by graph the polar coordinates first.



SAS △

use Law of cosines!! ☺

$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 5^2 + 8^2 - 2(5)(8)\cos 95°$

$x^2 = 95.97$

$\boxed{x = 9.8 \text{ miles}}$