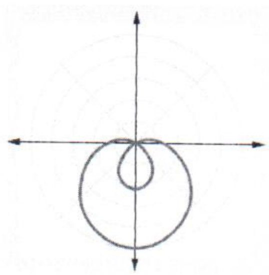
Secondary 3 Honors **Name:** ______________________

Notes 12-5: Complex Plane and Complex Numbers **Period:** ______

Warm-up:

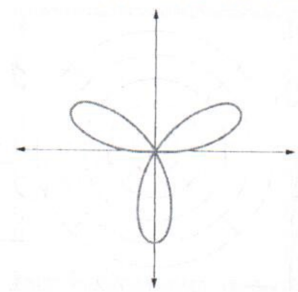
Identify the type of polar graph WITHOUT A CALCULATOR AND NOTES!

1.



Limacon
-inner loop

2.



Rose curve

Name the four types of polar graphs:

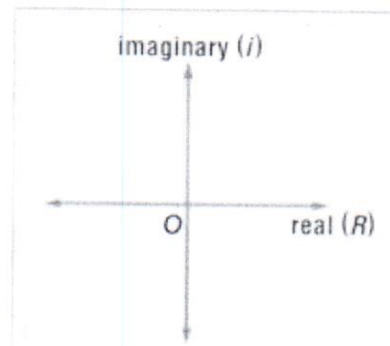
① circle ② Limacon ③ Rose curve ④ Lemniscate

COMPLEX NUMBERS

A complex number given in rectangular form $a+bi$ has a real component and an imaginary one.

Real component: a Imaginary Component: bi

You can graph a complex number on the complex plane by representing it with a point, (a, b). Just like a regular coordinate plane, we need two axes to graph a complex number. The real component is plotted on the horizontal axis, which we will now call the Real (R) axis. The imaginary component is plotted on the vertical axis, which we will now call the imaginary (i) axis.

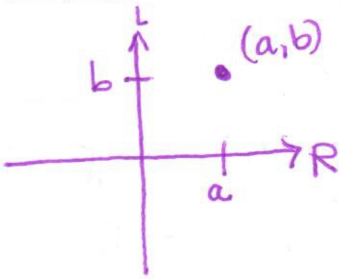


Complex Plane

Ex. 1: Graph the complex numbers.

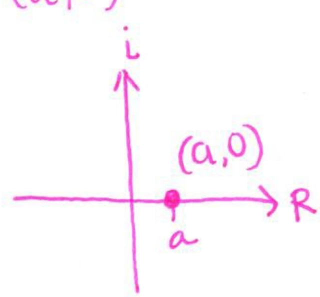
$y = a + bi$

(a, b)



$y = a + 0i$

(a, 0)



ABSOLUTE VALUE OF A COMPLEX NUMBER

Recall that the absolute value of a real number is the distance from zero on the number line.

It is similar for the absolute value of a complex number. It is the distance from zero in the complex plane. We solve the distance using the Pythagorean Theorem. $a^2+b^2=c^2$

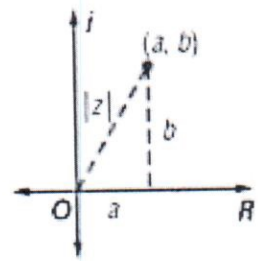
Absolute Value of a Complex Number

length of vector ⟹ abs value of complex #.

$z=a+bi$

$|z|^2=a^2+b^2$

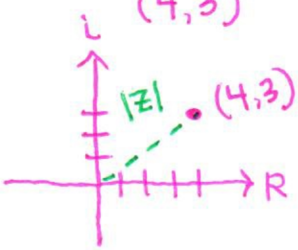
$\boxed{|z|=\sqrt{a^2+b^2}}$



Ex. 2: Graph each number in the complex plane, and find its absolute value.

a. $z=4+3i$

(4,3)



$|z|=\sqrt{4^2+3^2}$

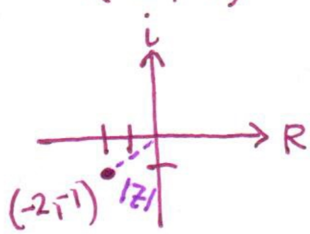
$=\sqrt{16+9}$

$=\sqrt{25}$

$\boxed{|z|=5}$

b. $z=-2-i$

(−2,−1)

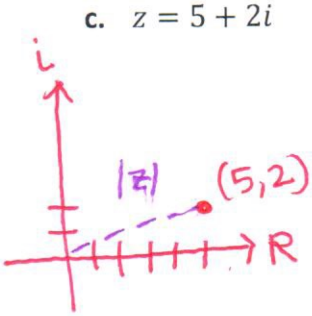


$|z|=\sqrt{(-2)^2+(-1)^2}$

$=\sqrt{4+1}$

$\boxed{|z|=\sqrt{5}}$

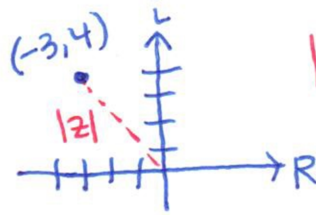
c. $z=5+2i$



$|z|=\sqrt{5^2+2^2}$

$\boxed{|z|=\sqrt{29}}$

d. $z=-3+4i$



$|z|=\sqrt{(-3)^2+4^2}$

$\boxed{|z|=5}$

CONVERTING TO POLAR FORM

Just like converting from rectangular coordinates to polar coordinates, we can convert from complex coordinates to polar coordinates.

Rect. $z=a+bi$

$z=r\cos\theta+(r\sin\theta)i$

$\boxed{\text{POLAR: } z=r(\cos\theta+i\sin\theta)}$

$a=r\cos\theta$

$b=r\sin\theta$

$r=|z|=\sqrt{a^2+b^2}$

$\theta=\tan^{-1}\left(\frac{b}{a}\right)$ IF $a>0$.

$\theta=\tan^{-1}\left(\frac{b}{a}\right)+\pi$

IF $a<0$.

Ex. 3: Express each complex number in polar form.

a. $-6+8i$ $\quad a<0$

$a \quad b$

b. $4+\sqrt{3}i$

$r = |z| = \sqrt{(-6)^2+8^2} = \sqrt{100} = 10$

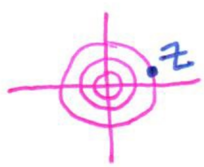
$\theta = \tan^{-1}\left(\frac{8}{-6}\right) + \pi = 2.21$

$= \boxed{10(\cos 2.21 + i\sin 2.21)}$

Ex. 4: Graph the complex polar equation and then convert it to rectangular form.

$$z = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$r = 3 \quad \theta = \frac{\pi}{6}$



$a+bi$

$z = 3(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$

$= 3\cos\frac{\pi}{6} + 3i\sin\frac{\pi}{6}$

$= 3\left(\frac{\sqrt{3}}{2}\right) + 3\left(\frac{1}{2}\right)i$

$\boxed{z = \frac{3\sqrt{3}}{2} + \frac{3}{2}i}$

PRODUCTS AND QUOTIENTS OF COMPLEX NUMBERS

We can multiply and divide complex numbers and use their product/quotient to then convert equations to other forms.

Given the complex numbers:

$$z_1 = r_1(\cos\theta_1 + i\sin\theta_1) \qquad z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$$

PRODUCT FORMULA

$$z_1 \cdot z_2 = r_1 \cdot r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$$

QUOTIENT FORMULA

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$$

Ex. 5: Find the product in polar form. Then express the answer in rectangular form.

$$2\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)\cdot 4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$

$$=(2)(4)\left[\cos\left(\frac{5\pi}{3}+\frac{\pi}{6}\right)+i\sin\left(\frac{5\pi}{3}+\frac{\pi}{6}\right)\right]$$

$$\frac{10\pi}{6}+\frac{\pi}{6}$$

$$=\boxed{8\left[\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right]}$$ polar form

$$8\cos\frac{11\pi}{6}+8i\sin\frac{11\pi}{6}$$

$$8\left(\frac{\sqrt{3}}{2}\right)+8\left(-\frac{1}{2}\right)i$$

$$\boxed{4\sqrt{3}-4i}$$ rect.

Ex. 6: If a circuit has a voltage, *E*, of 150 volts and an impedance, *Z*, of $6-3i$ ohms, find the current, *I*, (amps) in the circuit in rectangular form. Use $E = I\cdot Z$. $I=\frac{E}{Z}$

First, express each number in polar form.

★ $E = 150+0i$ $\quad$ $\theta=\tan^{-1}\left(\frac{0}{150}\right)$ $\quad$ $r=\sqrt{150^2+0^2}$

$a>0$ $\quad$ $\theta=0$ $\quad$ $r=150$

$$E=150(\cos 0+i\sin 0)$$

★ $Z=6-3i$ $\quad$ $\theta=\tan^{-1}\left(\frac{-3}{6}\right)$ $\quad$ $r=\sqrt{6^2+(-3)^2}$

$a>0$ $\quad$ $\theta=-0.46$ $\quad$ $r=\sqrt{45}$

$$Z=\sqrt{45}\left[\cos(-0.46)+i\sin(-0.46)\right]$$

$$\star\ I=\frac{E}{Z}=\frac{150}{\sqrt{45}}\left[\cos(0-(-0.46))+i\sin(0-(-0.46))\right]$$

$$=22.36\left[\cos 0.46+i\sin 0.46\right]$$

$$=22.36\cos 0.46+22.36i\sin 0.46$$

$$\boxed{I=20.04+9.93i \text{ amps}}$$