Unit 5: Radical Expressions Name:____________________

Secondary 3H: 5-3 Notes: Rational Exponents

Rewriting Radical Expressions with rational exponents:

- You can write a radical expression in an equivalent form using a rational exponent instead of a radical sign.

$$\rightarrow \sqrt[n]{x^{m}} = x^{m/n} \leftarrow$$

Rewrite each of the following in exponential form:

$\sqrt{36}$

$36^{1/2} = (6^2)^{1/2} = 6^{2/2} = 6$

$\sqrt[3]{64}$

$64^{1/3}$

$\sqrt[5]{x}$

$x^{1/5}$

Rewriting Exponential Expressions with radical signs:

- You can write an exponential expression in an equivalent form using a radical sign instead of a rational exponent.

$$\left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m} = x^{m/n}$$

Rewrite each of the following in radical form:

$x^{\frac{2}{3}}$

$\sqrt[3]{x^2}$ OR $(\sqrt[3]{x})^2$

$a^{\frac{1}{4}}$

$\sqrt[4]{a}$

- **Properties of Rational Exponents**

Property	Example
$a^m * a^n = a^{m+n}$	$8^{1/3} \cdot 8^{2/3} = 8^{1/3+2/3} = 8^{3/3} = 8$
$\frac{a^m}{a^n} = a^{(m-n)}$	$\frac{7^{3/2}}{7^{1/2}} = 7^{3/2-1/2} = 7^{2/2} = 7$
$(a^m)^n = a^{mn}$	$(5^{1/2})^4 = 5^{4/2} = 5^2 = 25$
$a^{-m} = \frac{1}{a^m}$	$9^{-1/2} = \frac{1}{\underset{\sqrt{9}}{9^{1/2}}} = \frac{1}{3}$
$(ab)^m = a^m b^m$	$(4 \cdot 5)^{1/2} = 4^{1/2} 5^{1/2} = 2 \cdot 5^{1/2}$ OR $2\sqrt{5}$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{27}\right)^{1/3} = \frac{5^{1/3}}{27^{1/3}} = \frac{\sqrt[3]{5}}{\sqrt[3]{27}} = \frac{\sqrt[3]{5}}{3}$

Simplifying rational expressions

$216^{\frac{1}{3}}$

$\sqrt[3]{216}$

$\boxed{6}$

$7^{\frac{1}{2}} * 7^{\frac{1}{2}}$

$7^{1/2+1/2}$

$\boxed{7}$

$5^{\frac{1}{4}} * 125^{\frac{1}{4}}$

$5^{1/4} * (5^3)^{1/4}$

$5^{1/4} \cdot 5^{3/4}$

$5^{1/4+3/4}$

$\boxed{5}$

$\sqrt{2} * \sqrt[3]{2}$

$2^{1/2} \cdot 2^{1/3}$

$2^{1/2+1/3}$

$2^{3/6+2/6} = 2^{5/6}$

$\sqrt[6]{2^5} = \boxed{\sqrt[6]{32}}$

$\sqrt{x^3} * \sqrt[3]{x^2}$

$x^{3/2} \cdot x^{2/3}$

$x^{3/2+2/3}$

$x^{9/6+4/6}$

$x^{13/6}$

$\sqrt[6]{x^{13}} = \boxed{x^2\sqrt[6]{x}}$

$\sqrt[3]{ab} \div \sqrt[4]{(ab)^2}$

$\frac{(ab)^{1/3}}{(ab)^{2/4}}$

$(ab)^{1/3-1/2}$

$(ab)^{2/6-3/6}$

$(ab)^{-1/6}$

$= \frac{1}{(ab)^{1/6}}$

$= \boxed{\frac{1}{\sqrt[6]{ab}}}$

Do we really understand??

Explain why $(-64)^{\frac{1}{3}} = -64^{\frac{1}{3}}$ but $(-64)^{\frac{1}{2}} \neq -64^{\frac{1}{2}}$.

Because you cannot take the square root of a negative number, but you can cube root a negative number.

Practice

- What is $\frac{\sqrt[4]{x^3}}{\sqrt[8]{x^2}}$ in simplest form?

$\frac{x^{3/4}}{x^{2/8}} = \frac{x^{3/4}}{x^{1/4}}$

$= x^{3/4-1/4} = x^{2/4}$

$= x^{1/2} = \boxed{\sqrt{x}}$

Write each expression in simplest form:

- $(-8x\sqrt{xy})^{\frac{2}{3}}$

$(-8x(xy)^{1/2})^{2/3}$

$(-8x \cdot x^{1/2} \cdot y^{1/2})^{2/3}$

$(-8x^{3/2}y^{1/2})^{2/3}$

$(-8)^{2/3}(x^{3/2})^{2/3}(y^{1/2})^{2/3}$

$(-8)^{2/3}xy^{1/3}$

$\sqrt[3]{(-8)^2}$

$\sqrt[3]{64}$

$\boxed{4x\sqrt[3]{y}}$

- $(16y^8)^{-\frac{3}{4}}$

$\frac{1}{(16y^8)^{3/4}}$

$\frac{1}{16^{3/4}(y^8)^{3/4}}$

$\frac{1}{16^{3/4}y^{24/4}}$

$\frac{1}{\sqrt[4]{16^3}y^6}$

$\boxed{\frac{1}{8y^6}}$

- $\sqrt[3]{2} * \sqrt[5]{4}$

$2^{1/3} \cdot 4^{1/5}$

$2^{1/3} \cdot (2^2)^{1/5}$

$2^{1/3} \cdot 2^{2/5}$

$2^{1/3+2/5}$

$2^{5/15+6/15}$

$2^{11/15}$

$\sqrt[15]{2^{11}}$

$\boxed{\sqrt[15]{2048}}$