

## Secondary 3H: 5-4 Notes: Solving Radicals

Isolating the Radical

$$\begin{array}{r} 3 + \sqrt{2x-3} = 8 \\ -3 \quad -3 \end{array}$$

$$\frac{(\sqrt{2x-3})^2 = (5)^2}{}$$

$$\begin{array}{r} 2x-3 = 25 \\ +3 \quad +3 \end{array}$$

$$\frac{2x = 28}{2 \quad 2}$$

$$\boxed{x=14}$$

$$\begin{array}{r} \sqrt{4x+1} - 5 = 0 \\ +5 \quad +5 \end{array}$$

$$\frac{(\sqrt{4x+1})^2 = (5)^2}{}$$

$$\begin{array}{r} 4x+1 = 25 \\ -1 \quad -1 \end{array}$$

$$\frac{4x = 24}{4 \quad 4}$$

$$\boxed{x=6}$$

$$3\sqrt{x} + 3 = 15$$

$$\begin{array}{r} -3 \quad -3 \end{array}$$

$$\frac{3\sqrt{x} = 12}{3 \quad 3}$$

$$(\sqrt{x})^2 = (4)^2$$

$$\boxed{x=16}$$

Solving with Exponents

- How can I solve an equation in rational exponent form?

$$\bullet 3(x+1)^{\frac{2}{3}} = 12$$

- Divide by 3

- Get rid of the exponent

- subtract 1

- To solve equations of the form  $x^{\frac{m}{n}} = k$ , raise each side of the equation to the power  $\frac{n}{m}$ , the reciprocal of  $\frac{m}{n}$ . If either  $m$  or  $n$  is

even, then  $(x^{\frac{m}{n}})^{\frac{n}{m}} = |x|$ .

- For example,

$$\frac{3(x+1)^{\frac{2}{3}} = 12}{3 \quad 3}$$

$$\frac{((x+1)^{\frac{2}{3}})^{\frac{3}{2}} = (4)^{\frac{3}{2}}}{}$$

$$|x+1| = 4^{\frac{3}{2}}$$

$$|x+1| = \sqrt{4^3} \left\{ \begin{array}{l} 4 \\ 4 \end{array} \right.$$

$$|x+1| = 4\sqrt{4}$$

$$|x+1| = 8$$

$$\begin{array}{l} x+1=8 \\ \boxed{x=7} \end{array}$$

$$\begin{array}{l} x+1=-8 \\ \boxed{x=-9} \end{array}$$

$$\frac{2(x+3)^{\frac{2}{3}} = 8}{2 \quad 2}$$

$$\frac{((x+3)^{\frac{2}{3}})^{\frac{3}{2}} = (4)^{\frac{3}{2}}}{}$$

$$|x+3| = 4^{\frac{3}{2}}$$

$$|x+3| = 8$$

$$x+3=8$$

$$\boxed{x=5}$$

$$x+3=-8$$

$$\boxed{x=-11}$$

▪ Earth Science Example

- For Meteor Crater in Arizona, the formula  $d = 2 \sqrt[3]{\frac{V}{0.3}}$  relates the diameter  $d$  of the rim (in meters) to the volume  $V$  (in cubic meters). What is the volume of Meteor Crater? (All values are approximate.)



• Hint: There are 1000 meters in a kilometer.

$1.2 \text{ km} = 1200 \text{ m}$

$$\frac{d}{2} = \frac{2 \sqrt[3]{\frac{V}{0.3}}}{2} \quad V = \frac{0.3d^3}{8}$$

$$\left(\frac{d}{2}\right)^3 = \left(\sqrt[3]{\frac{V}{0.3}}\right)^3 \quad V = \frac{0.3(1200)^3}{8}$$

$0.3 \cdot \frac{d^3}{2^3} = \frac{V}{0.3} \cdot 0.3$   $V = 64,800,000 \text{ m}^3$

▪ Extraneous Solutions AGAIN!!!

- Raising both sides of an equation to a POWER can introduce solutions of the modified equation that are not solutions of the original equation.

$$\begin{aligned} \sqrt{x+7} - 5 &= x \\ +5 \quad +5 \\ \hline (\sqrt{x+7})^2 &= (x+5)^2 \\ x+7 &= (x+5)(x+5) \\ x+7 &= x^2+10x+25 \\ -x \quad -7 \quad -x \quad -7 \\ \hline 0 &= x^2+9x+18 \\ 0 &= (x+3)(x+6) \end{aligned}$$

ex. sol.  $x = -6$   $x = -3$

$$\begin{aligned} \sqrt{-6+7} - 5 &\stackrel{?}{=} -6 \\ -4 &\neq -6 \\ \sqrt{-3+7} - 5 &\stackrel{?}{=} -3 \\ -3 &= -3 \end{aligned}$$

$$\begin{aligned} \sqrt{x-5} &= -2 \\ (\sqrt{x-5})^2 &= (-2)^2 \\ x-5 &= (-2)^2 \\ |x-5| &= 4 \\ x-5 &= 4 \quad x-5 = -4 \\ x &= 9 \quad \text{OR} \quad x = 1 \end{aligned}$$

Got it?

$$\begin{aligned} \sqrt{5x-1} + 3 &= x \\ -3 \quad -3 \\ \hline (\sqrt{5x-1})^2 &= (x-3)^2 \\ 5x-1 &= (x-3)(x-3) \\ 5x-1 &= x^2-6x+9 \\ -5x+1 \quad -5x+9 \\ \hline 0 &= x^2-11x+10 \end{aligned}$$

$$0 = (x-10)(x-1)$$

$x = 10$   $x = 1$  ex. sol.

$$\begin{aligned} \sqrt{5(10)-1} + 3 &\stackrel{?}{=} 10 \\ \sqrt{5(1)-1} + 3 &\stackrel{?}{=} 1 \quad \times \end{aligned}$$



Two Radicals in the same equation...

Isolate the more COMPLEX radical.

sqrt(2x+1) - sqrt(x) = 1

sqrt(2x+1) + sqrt(x) = 1 + sqrt(x)
(2x+1) = (1+sqrt(x))^2

2x+1 = (1+sqrt(x))(1+sqrt(x))

2x+1 = 1 + 2sqrt(x) + x

(x)^2 = (2sqrt(x))^2

x^2 = 4x
-4x -4x

x^2 - 4x = 0

x(x-4) = 0

x = 0, x = 4

sqrt(2(0)+1) - sqrt(0) = 1

sqrt(2(4)+1) - sqrt(4) = 1

sqrt(5x+4) - sqrt(x) = 4

(sqrt(5x+4))^2 = (4+sqrt(x))^2

5x+4 = (4+sqrt(x))(4+sqrt(x))

5x+4 = 16 + 8sqrt(x) + x

(4x-12)^2 = (8sqrt(x))^2

(4x-12)(4x-12) = 64x

16x^2 - 96x + 144 = 64x

16x^2 - 160x + 144 = 0

16(x^2 - 10x + 9) = 0

16(x-9)(x-1) = 0

x = 9, x = 1 (ex. sol.)

sqrt(5(9)+4) - sqrt(9) = 4

sqrt(5(1)+4) - sqrt(1) = 4

sqrt(3-x) + sqrt(x+2) = 3

(sqrt(3-x))^2 = (3 - sqrt(x+2))^2

3-x = (3 - sqrt(x+2))(3 - sqrt(x+2))

3-x = 9 - 3sqrt(x+2) - 3sqrt(x+2) + x+2

3-x = 11 + x - 6sqrt(x+2)

(-8-2x)^2 = (-6sqrt(x+2))^2

(-8-2x)(-8-2x) = 36(x+2)

64 + 16x + 16x + 4x^2 = 36x + 72

4x^2 - 4x - 8 = 0

4(x^2 - x - 2) = 0

4(x+1)(x-2) = 0

x = -1, x = 2

sqrt(3+1) + sqrt(-1+2) = 3

sqrt(3-2) + sqrt(2+2) = 3

64 + 32x + 4x^2 = 36x + 72
-36x -72 -36x -72