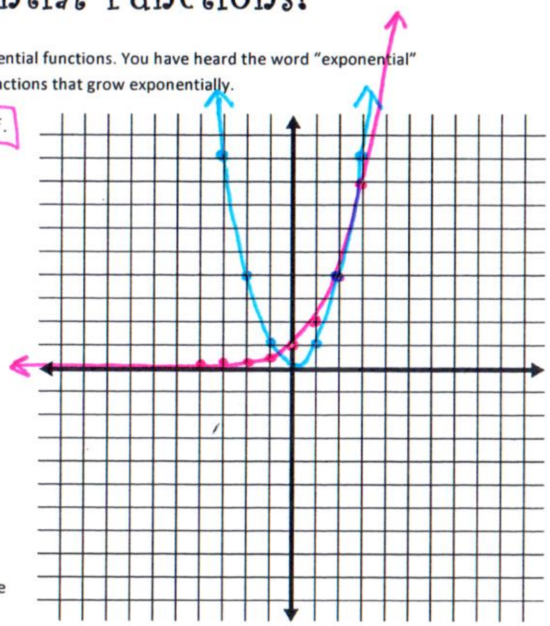
Exponential Functions!

Today we are going to learn about exponential functions. You have heard the word "exponential" before. Today we are going to explore functions that grow exponentially.

Begin by considering the function $y = 2^x$.
Make a table of values for this function:

x	y
-4	1/16
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8
4	16
5	32

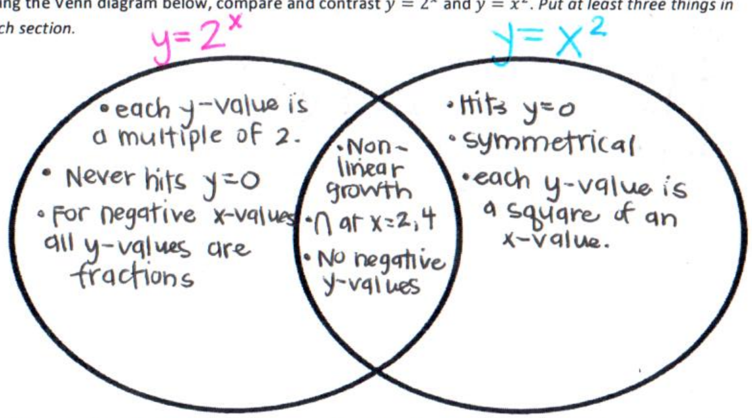


Now using this table of values, graph the function by hand.

On the same graph, draw $y = x^2$. How are the two graphs similar? How are they different?

Both grow on R. one levels out.

Using the Venn diagram below, compare and contrast $y = 2^x$ and $y = x^2$. *Put at least three things in each section.*



When you are finished, share what you have with the person sitting next to you. Add anything they have that you don't.

The functions $y = x^2$ and $y = 2^x$ each involve a base raised to a power, but the roles are reversed:

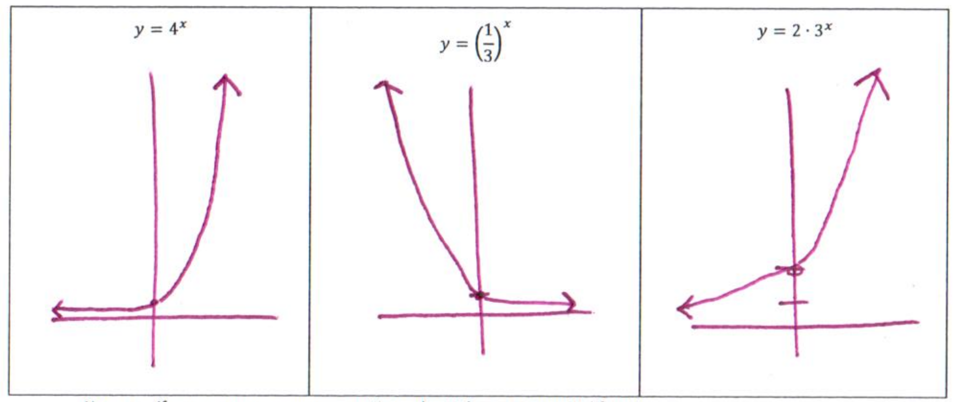
- For $y = x^2$, the base is the variable x, and the exponent is the constant 2.
- For $y = 2^x$, the base is the constant 2, and the exponent is the variable x. This is an *exponential function*.

Exponential Functions:

Exponential functions are functions which represent repeated multiplication. For example, $y = 2^x$ represents multiplying 2 by itself x number of times. They are defined and continuous for all real numbers.

They are written in the form $f(x) = a \cdot b^x$, where a is nonzero, b is positive, and $b \neq 1$.

Let's explore some different exponential functions! For each function below, use your calculator to find the graph of the function. Then sketch this graph in the box provided.



Now, see if you can answer some questions about these exponential functions:

- Will these graphs ever hit $y = 0$? Why or why not?
  No, because $b^x \neq 0$.
- Will these graphs ever have negative y values? Why or why not?
  No, because negative exponents make fractions not negative numbers.
- Remember that exponential functions are in the form $y = a \cdot b^x$. What does a do?
  a is the y-intercept.
- What if b is a fraction less than 1? Why does it look the way it does?
  Starts up on the left
  decreasing.

For exponential functions, $y = a \cdot b^x$, a is the initial value of the function, or in other words, the y-intercept. b is the base, which is the number being multiplied repeatedly.

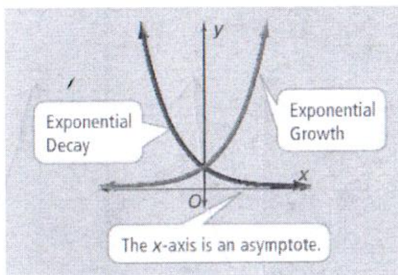
Unless the graph is shifted vertically, the graph will never hit $y = 0$ because there is no value of x for which $b^x = 0$. Not even when $x = 0$! (If you don't believe me, let $x = 0$ and see what you get!) Therefore, we would say that $y = 0$ is the horizontal asymptote.

Unless the graph is shifted vertically or reflected, the graph will never have negative y values. This is because there is no exponent x for which a number b^x will be negative. Not even when x is negative! (If you don't believe, try letting x be negative and see what happens. Was b^x negative? Nope!)

What about when b is a fraction less than 1? Well . . . there are two types of exponential behavior: exponential growth and exponential decay.

For exponential growth, as the value of x increases, the value of y increases. For exponential decay, as the value of x increases, the value of y decreases, approaching zero. The exponential functions shown here are asymptotic to the x-axis.

So what determines whether the exponential function will be growing or decaying? b!



For the function $y = a \cdot b^x$,

- If $a > 0$ and $b > 1$, the function represents exponential growth. b is known as the growth factor.
- If $a > 0$ and $0 < b < 1$, (or in other words b is a fraction less than 1) the function represents exponential decay. b is known as the decay factor.

In either case, the y-intercept is $(0, a)$, the domain is all real numbers, the asymptote is $y = 0$, and the range is $y > 0$.

For each of the following functions or situations, identify whether it is an example of exponential growth or decay. Then find the y-intercept.

1. $y = 3(4^x)$ growth, y-int: 3
2. $y = 11(0.75^x)$ decay, y-int: 11
3. You put \$2000 into a college savings account for four years. The account pays 6% interest annually. growth, y-int: \$2000
4. $y = 12\left(\frac{17}{10}\right)^x$ growth, y-int: 12
5. $y = 4\left(\frac{5}{6}\right)^x$ decay, y-int: 4

Writing an exponential function:

Often, we are given a limited amount of information or a table of values and asked to find the exponential function that models this data. In order to write a function of the form $y = a \cdot b^x$, you need to find the growth or decay factor b. If you know y-values for two consecutive x-values, you can find b. Or if you know the initial value a and another point (x, y), you can find b.

Let's look at a couple of examples:

Using a table of values

Keep in mind that exponential growth is repeated multiplication. Therefore if we can find what factor you multiplied by to get from one y-value to another, we can find b. Look at the example below to see what I mean.

Table 3.2 Values for Two Exponential Functions

x	$g(x)$		$h(x)$	
−2	4/9		128	
		× 3		$\times \frac{1}{4}$
−1	4/3		32	
		× 3		$\times \frac{1}{4}$
0	4		8	
		× 3		$\times \frac{1}{4}$
1	12		2	
		× 3		$\times \frac{1}{4}$
2	36		1/2	

Notice that for $g(x)$ each y-value is multiplied by a factor of 3 to get to the next y-value. Therefore, we would say that the growth factor (or b) is 3. Then $g(x) = a \cdot 3^x$. To find a we just look at what the value of y is when $x = 0$, since this would be the y-intercept (or initial value). Then the equation for $g(x)$ is $g(x) = 4 \cdot 3^x$.

Can you find the equation for $h(x)$?

x	y
0	8
1	2
2	1/2

$y = ab^x$

$h(x) = 8\left(\frac{1}{4}\right)^x$

Using a situation

In 2009, there were 1570 bears in a wildlife refuge. In 2010, the population had increased to approximately 1884 bears. If this trend continues and the bear population is increasing exponentially, how many bears will there be in 2018?

Since 2009 is the first population we have, we let this be the initial population. Therefore in our equation $y = a \cdot b^x$, $a = 1570$. Then our equation so far is $y = 1570(b^x)$. In order to find b, we need to use the other information we have. We know that in 2010, the population was 1884. Since 2010 is 1 year after 2009 $(2010 - 2009 = 1)$, then we let $x = 1$ and $y = 1884$. Then we solve for b.

$$1884 = 1570(b^1)$$

$$\frac{1884}{1570} = b$$

$$1.2 = b$$

So our final equation is $y = 1570\ (1.2^x)$. Now we just need to find the population in 2018. Since 2018 is 9 years after 2009 $(2018 - 2009)$, then we plug 9 in for x and solve for y.

$$y = 1570(1.2^9)$$

$$y = 8100.86$$

The population of bears in 2018 will be approximately 8100.

Now you try!

48in

1. Your friend drops a rubber ball from 4 ft. You notice that its rebound is 32.5 in. on the first bounce and 22 in. on the second bounce.
   a) What exponential function would be a good model for the height of the ball?
   b) How high will the ball bounce on the fourth bounce?

x	y
0	48 in
1	32.5
2	22

a) $y = a \cdot b^x$

$32.5 = 48 \cdot b^1$

$b = 0.677$

$\boxed{y = 48(0.677)^x}$

b) $x = 4$

$y = 48(0.677)^4$

$\boxed{y = 10.08 \text{ in}}$

2. A music store sold 200 guitars in 2007. The store sold 180 guitars in 2008. The number of guitars that the store sells is decreasing exponentially. If this trend continues, how many guitars will the store sell in 2012?

x	y
0	200
1	180
5	?

$y = 200 \cdot b^x$

$180 = 200 \cdot b^1$

$b = 0.9$

$y = 200(0.9)^5$

$\boxed{y = 118 \text{ guitars}}$

Once you have finished this packet, the rest of the time is yours to work on your homework.