Secondary 3H: 6-3 Notes: Logarithms

Warm-Up:

- A scientist notes the bacteria count in a Petri dish is 40. Three hours later, she notes the count has increased to 75. How many hours will it take for the bacteria count to grow to 120? Express the answer to the nearest tenth of an hour.

x	y
0	40
3	75
?	120

$75 = 40b^3$

$\sqrt[3]{b^3} = \sqrt[3]{\frac{75}{40}}$

$b = 1.233$

$120 = 40(1.233)^x$

$\boxed{x = 5.2 \text{ hours}}$

Logarithms

- $\frac{9000}{3000} = \frac{3000e^{.05t}}{3000}$ How do we solve for t?

  $3 = e^{0.05t}$

- What if it were . . . ?
  - $\frac{9000}{3000} = \frac{\cancel{3000}x}{\cancel{3000}}$ $x = 3$
  - $9000 - 3000 = \cancel{3000} - 3000 - x$ $\frac{-x}{-1} = \frac{+6000}{-1}$ $x = -6000$
  - $3000 \cdot 9000 = \frac{x}{\cancel{3000}} \cdot \cancel{3000}$

    $x = 27,000,000$

Logarithms are the inverses of exponentials!!! $y = b^x$

- A logarithm base b of a positive number x satisfies the following definition: For $\boxed{b > 0}$, $b \neq 1$,

$$\text{If } b^y = x, \text{ then } \log_b x = y.$$

log base b of x equals y.

- *Remember that an inverse "undoes" what was done!*
- So if $2^3 = 8$, then $\log_2 8 = 3$

Quick Write!

- Why can't $b = 1$?
- **(Remember that $b^y = x \to \log_b x = y$)**

Because $1^{\text{any power}} = 1$ and the log properties don't apply.

Using the Definition

- Write each of the following in logarithmic form:
  - $10^1 = 10$

    $\log_{10} 10 = 1$
  - $4^2 = 16$

    $\log_4 16 = 2$
  - $289^{\frac{1}{2}} = 17$

    $\log_{289} 17 = \frac{1}{2}$
  - $3^{-2} = \frac{1}{9}$

    $\log_3 \frac{1}{9} = -2$

- You are NOT allowed to use a calculator on the following worksheet.
- Your job is to place each expression on the number line according to what it is equal to. Then MAKE SURE to explain how you knew where to place what!
- You may have to estimate on a couple of them!
- Remember:

  $If\ x = b^y\ then\ \log_b x = y$

- **Your Turn!**
  - Write each of the following in logarithmic form:
    - $36 = 6^2$ $\quad \log_6 36 = 2$
    - $\frac{8}{27} = \left(\frac{2}{3}\right)^3$ $\quad \log_{\frac{2}{3}} \frac{8}{27} = 3$
    - $10^{-2} = .01$ $\quad \log_{10} 0.01 = -2$

- **Evaluating a Logarithm**
  - What is the value of $\log_8 32$?

$\log_8 32 = x$

$8^x = 32$

$(2^3)^x = 2^5$

$2^{3x} = 2^5$

$3x = 5$ $\boxed{x = \frac{5}{3}}$

- **Your turn**
  - What is the value of $\log_4 32$?

$4^x = 32$

$(2^2)^x = 2^5$ $2x = 5$ $\boxed{x = 5/2}$

  - What is the value of $\log_{64}\left(\frac{1}{32}\right)$?

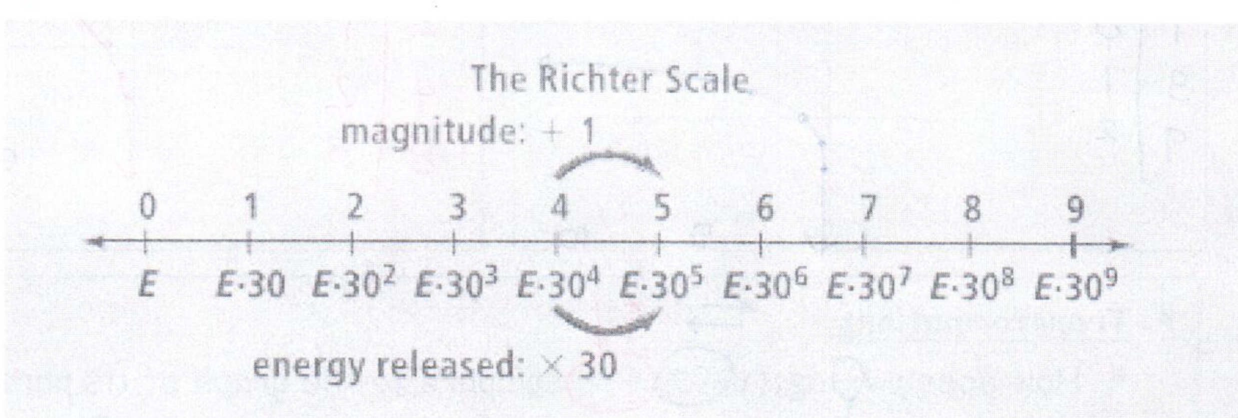
$64^x = \frac{1}{32}$

$64^x = 32^{-1}$ $6x = -5$

$(2^6)^x = 2^{-5}$ $\boxed{x = -5/6}$

- **Common Logs**
  - A logarithm base 10 is frequently used, so to make it quicker to write, we can leave out the 10! $\log_{10} x = \log x$

- **Logarithmic Scale**
  - Many measurements of physical phenomena have such a wide range of values that the reported measurements are logarithms (exponents) of the values, not the values themselves.
  - When you use the logarithm of a quantity instead of the quantity itself, you are using a logarithmic scale!
  - For an increase of 1 on the Richter Scale, the amplitude of the ground motion recorded by the seismograph increases by a factor of ten. CRAZY . . .



- **EARTHQUAKE!!!!**
  - In December 2004, an earthquake with magnitude 9.3 on the Richter scale hit off the northwest coast of Sumatra. The magnitude of an earthquake that hit Sumatra in March 2005 was 8.7. The formula $\boxed{\log\left(\frac{I_1}{I_2}\right) = M_1 - M_2}$ compares the intensity levels of earthquakes where I is the intensity level determined by a seismograph, and M is the magnitude on a Richter scale. How many times more intense was the December earthquake than the March earthquake?

$\log\left(\frac{I_1}{I_2}\right) = 9.3 - 8.7$

$\log\left(\frac{I_1}{I_2}\right) = 0.6$

$10^{0.6} = \frac{I_1}{I_2} = 3.98\ldots$

Dec EQ was about 4x more intense than March EQ

- **Your Turn:**
  - In 1995, an earthquake in Mexico registered 8.0 on the Richter scale. In 2001, an earthquake of magnitude 6.8 shook Washington State. How many times more intense was the 1995 earthquake than the 2001 earthquake?

$\log\left(\frac{I_1}{I_2}\right) = 8.0 - 6.8$

$\log\left(\frac{I_1}{I_2}\right) = 1.2$

$10^{1.2} = \frac{I_1}{I_2} = 15.8$

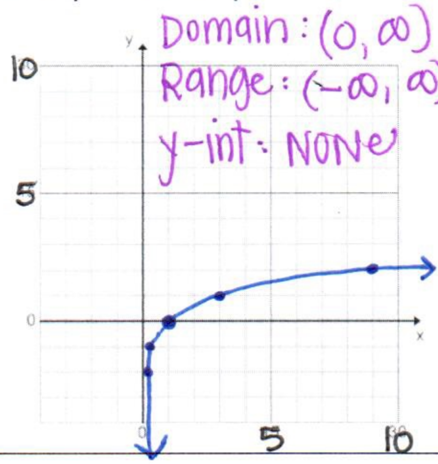
Mexico earthquake was about 16x more intense than U.S.

- **Graphing Logarithms**
  - What is the graph of $y = \log_3 x$? Describe the domain, range, and y-intercept.

$3^y = x$

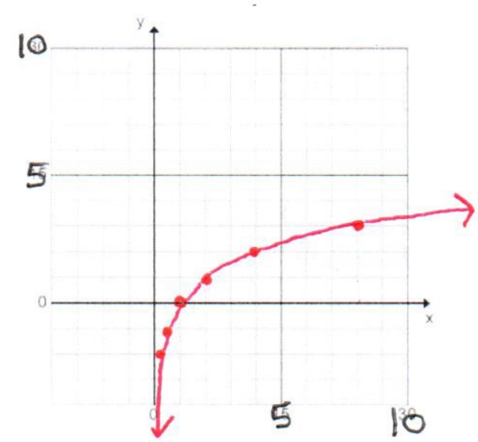
x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
y-int: NONE



- **Your Turn**
  - Graph $y = \log_2 x \Rightarrow 2^y = x$

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3



- **Transformations**
  - How does $y = \log_4(x - 3) + 4$ compare to the graph of its parent function (which is $y = \log_4 x$)?

up 4, Right 3

negative: reflect

stretch/compress