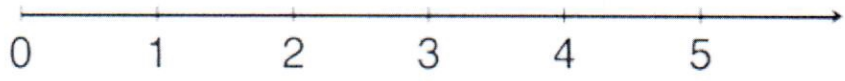
Secondary 3H: 6-4 Notes: Properties of Logarithmic Functions

Warm-up:
WITHOUT USING A CALCULATOR, Place each expression on the number line. You may need to estimate.

A. $\log_{10}50$ B. $\log_{10}150$ C. $\log_{10}1000$ D. $\log_{10}500$



Properties of Logarithms

Logarithms and Exponents have corresponding properties! To figure out the properties of logarithms, we can use the properties of exponents.

- Remember that $log_b b^k = k$
- Let $x = log_b m$ and $y = log_b n$
  - What does m equal?
    $m = b^x$
  - What does n equal?
    $n = b^y$
  - Find $mn = (b^x)(b^y) = b^{x+y}$
  - Find $\frac{m}{n} = \frac{b^x}{b^y} = b^{x-y}$

THREE LOGARITHM PROPERTIES

- Product Property:
  $log_b mn = \log_b m + \log_b n$
- Quotient Property:
  $log_b \frac{m}{n} = \log_b m - \log_b n$
- Power Property:
  $log_b m^n = n log_b m$

Simplifying

- $log_4 32 - log_4 2$
  $\log_4 \frac{32}{2} = \log_4 16 = \boxed{2}$
- $6log_2 x + 5log_2 y$
  $\log_2 x^6 + \log_2 y^5$
  $\boxed{\log_2 x^6 y^5}$

Your Turn!!

- $log_4 5x + log_4 3x$
  $\log_4 (5x)(3x) = \boxed{\log_4 15x^2}$
- $2log_4 6 - log_4 9$
  $\log_4 6^2 - \log_4 9$
  $\log_4 36 - \log_4 9$
  $\log_4 \frac{36}{9} = \log_4 4 = \boxed{1}$

Expanding

- $\log_b\left(\frac{x^4}{729}\right)$ ← = $\log_b x^4 - \log_b 729$

  $4\log_b x - \log_b 729$

- $\log \frac{4x}{y}$ ← $\log 4x - \log y$

  $\log 4 + \log x - \log y$

Your Turn!

- $\log_3\left(\frac{250}{37}\right)$ = $\log_3 250 - \log_3 37$

- $\log_3(7(2x-3)^2)$ = $\log_3 7 + \log_3 (2x-3)^2$

  $\log_3 7 + 2\log_3 (2x-3)$

Change of Base

Unless you have a fancy calculator, it will only use log base 10. So we can use the following formula to evaluate a logarithm with any base.

* c is usually 10.

$$\log_b m = \frac{\log_c m}{\log_c b}$$

b: base; b: basement

Two Methods to solve

What is the value of $\log_{81} 27$

Method 1: Use a common base!
(But...this requires a little more work)

$81^x = 27$

$(3^4)^x = 3^3$

$4x = 3$

$x = 3/4$

Method 2: Use a calculator.

$\log_{81} 27 = \frac{\log 27}{\log 81}$

$= \frac{3}{4}$

Your Turn!
Find the value of each expression using whichever method you prefer.

- $log_8 32$ $\frac{\log 32}{\log 8}$ OR $8^x = 32$, $(2^3)^x = 2^5$, $3x = 5$, $x = 5/3$
- $log_{12} 20$ $\frac{\log 20}{\log 12}$ OR $12^x = 20$ NOT EASY! $x = 1.21$

pH levels
The pH of a substance equals $-log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions. $[H^+{}_a]$ for household ammonia is 10^{-11}. $[H^+{}_v]$ for vinegar is 6.3×10^{-3}. What is the difference for pH levels of ammonia and vinegar?

$-\log(H^+_a) - (-\log(H^+_v))$

$-\log(10^{-11}) - \log(6.3 \times 10^{-3})$

$11 \log 10 + \log 6.3 + \log 10^{-3}$

$11 \log 10 + \log 6.3 - 3 \log 10$

$8 \log 10 + \log 6.3$

$8 + \log 6.3 = 8 + 0.8 = 8.8$

Your Turn
The concentration of hydrogen ions in household dish detergent is 10^{-12}. What is the pH level of household dish detergent?

$= -\log(10^{-12})$

$= 12 \log 10$

$12(1)$

$= 12$