

Secondary 3H 6-5 Notes: Solving Exponential and Logarithmic Equations

Determine if each expression is true or false. Justify your answer.

TRUE $\log 12 = \log 4 + \log 3$
 $\log 12 = \log(4 \times 3)$
 $\log 12 = \log 12$

$\log\left(\frac{3}{5}\right) = \frac{\log 3}{\log 5}$ FALSE
 $\log 3 - \log 5 \neq \frac{\log 3}{\log 5}$

TRUE $\log_6 12 + \log_6 3 = 2$
 $\log_6(12 \times 3)$
 $\log_6 36 = 2 \checkmark$

$\frac{1}{2} \log_4 4x = \log_4 2x$ FALSE
 $\log_4 (4x)^{1/2} = \log_4 \sqrt{4x} = \log_4 2\sqrt{x} \neq \log_4 2x$

YOU'RE A WINNER!

You are the winner of a very special TV game show! You are given the choice of one of two prizes:

1) \$100 a week $y = 100x$

2) \$1 this week, \$2 next week, \$4 the third week, and so on (so that the amount you receive doubles every week).

$$\begin{array}{r} x \ y \\ 1 \ 1 \\ 2 \ 2 \\ 3 \ 4 \\ \hline \end{array}$$

$y = 2^{x-1}$

Which prize would you choose and why?

At week 7, which prize is better?

$x = 7$
 $y = 100(7)$
 \$700

$y = 2^{7-1}$
 = \$64

FIRST PRIZE

At week 20, which prize is better?

$x = 20$
 $y = 100(20)$
 \$2000

$y = 2^{20-1}$

= \$524,288

SECOND PRIZE

How long will it take each prize to reach \$1,000,000?

x $1000000 = 100x$
 $x = 10,000$ weeks
 192 years

y $1000000 = 2^{x-1}$
 $\log_2 1000000 = x-1$
 $x = 20.9$ weeks

almost 5 months

When two numbers in an exponential equation have common bases, we can find the answer easily.

What is the solution to $16^{3x} = 8$?

$(2^4)^{3x} = 2^3$

$12x = 3$

$x = \frac{3}{12} = \boxed{\frac{1}{4}}$

But what if the bases are NOT common?

$$\log(15^{3x}) = \log(285)$$

$$\log 15^{3x} = \log 285$$

$$\frac{3x \log 15}{\log 15} = \frac{\log 285}{\log 15}$$

$$3x = \frac{\log 285}{\log 15} \quad \frac{3x}{3} = \frac{2.087}{3}$$

$$x = 0.696$$

Your Turn!

$$5 - 3^x = -40$$

$$-5 - 3^x = -45$$

$$-3^x = -45$$

$$3^x = 45$$

$$\log 3^x = \log 45$$

$$x \log 3 = \log 45$$

$$x = \frac{\log 45}{\log 3}$$

$$x = 3.46$$

Does this apply to our prize money?

$$1,000,000 = 100x$$

$$1,000,000 = 2^x$$

Resource Management

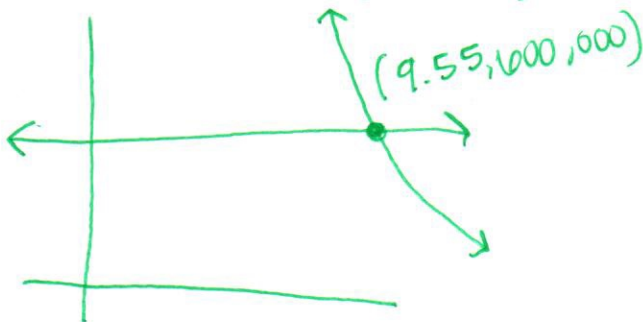
- Wood is a sustainable, renewable, natural resource when you manage forests properly. Your lumber company has 1,200,000 trees. You plan to harvest 7% of the trees each year. How many years will it take to harvest half of the trees?

$$y = 600,000$$

$$y = a(1+r)^t$$

Use a graph to make a prediction and then check your prediction using logarithms.

$$600,000 = 1,200,000 (1-0.07)^t$$



$$\frac{600,000}{1,200,000} = \frac{1,200,000 (1-0.07)^t}{1,200,000}$$

$$0.5 = 0.93^t$$

$$\log_{0.93} 0.5 = t$$

$$\frac{\log 0.5}{\log 0.93} = t = 9.55 \text{ years}$$

Group Work

How do we solve logarithms?

As a group, come up with a plan to solve the following equation. See if your plan actually works.

- ① Find base of log.
- ② change log \rightarrow exp.
- ③ simplify exp.
- ④ Add like terms
- ⑤ Get x by itself
by opposite operations
- ⑥ enjoy your x ! 😊
- ⑦ check your answer

$$\log(4x - 3) = 2$$

$$10^2 \Rightarrow 100 = 4x - 3$$

$$\frac{103}{4} = \frac{4x}{4}$$

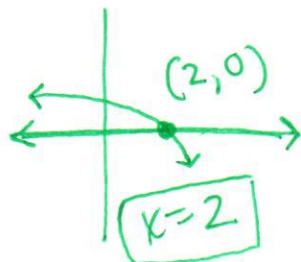
$$x = 25.75$$

- ① Get a calculator that graphs
- ② Turn it on.
- ③ $y =$
- ④ $y_1 = 2$
 $y_2 = \log(4x - 3)$
- ⑤ Adjust window
- ⑥ graph
- ⑦ 2nd | trace
5: Intersect
- ⑧ write down x .
- ⑨ check answer

Does our plan *always* work?

- $\log(5 - 2x) = 0$

$$\begin{array}{l} 10^0 = 5 - 2x \\ 1 = 5 - 2x \\ -5 - 5 \\ \hline -4 = -2x \\ -2 \quad -2 \\ \hline x = 2 \end{array}$$



- $2 \log(x + 1) = 5$

How can Logarithmic Properties help?

- $\log(x - 3) + \log(x) = 1$

$$\log(x(x-3)) = 1$$

$$\log(x^2 - 3x) = 1$$

$$10^1 = x^2 - 3x$$

$$0 = x^2 - 3x - 10$$

$$0 = (x-5)(x+2)$$

$$\boxed{x=5} \quad x=-2 \text{ ex. sol.}$$