**Secondary 3H 6-6 Notes: Natural Logarithms**

## Warm-up

1. Write a logarithmic equation that needs to be solved using the properties of logarithms. E.g. $y = \log 6 - \log 3x - 2$
2. Pass the logarithmic equation to your partner to solve.
3. Once they have solved it, check their answer. Help them fix any mistakes.

Equation:

$y = \log 6 - \log 3x - 2$

$y = \log \frac{6}{3x} - 2$

Solution:

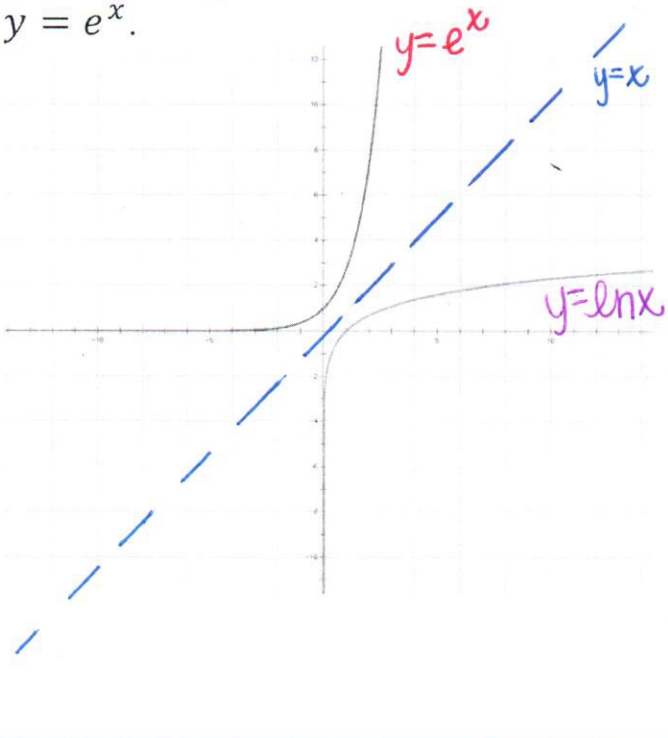
WITH YOUR PARTNER!

## When did the Cougar die?

As a group, use your investigative skills to determine if the hunters were being honest!

## Natural Logarithms

A natural logarithm is the inverse of $y = e^x$.




## Natural Logarithms

Because natural logarithms are still logarithms (just with a different base), then all of the same logarithm properties still apply.

- $2\ln(15) - \ln(75)$

  $\ln 15^2 - \ln 75$

  $\ln\left(\frac{15^2}{75}\right) = \ln\frac{225}{75} = \ln 3$

- $\ln(9) + \ln(12)$

  $\ln(9 \cdot 12)$

  $\ln 108$

## Solving Natural Logarithms

- $\ln(x-3)^2 = 4$

$\frac{\cancel{2}\ln(x-3)}{\cancel{2}} = \frac{4}{2}$

$\ln(x-3) = 2$

$e^2 = (x-3)$

$+3 \qquad +3$

$x = e^2 + 3$ $\boxed{x \approx 10.4}$

## Solving Exponentials

- $4e^{2x} + 2 = 16$

$-2 \qquad -2$

$\frac{4e^{2x}}{4} = \frac{14}{4}$

$e^{2x} = \frac{7}{2}$

$\frac{\ln\frac{7}{2}}{2} = \frac{2x}{2}$

$x = \frac{1}{2}\ln\frac{7}{2}$

$\boxed{x \approx 0.63}$

## Your Turn!

- $\ln(2x) + \ln(3) = 2$

$\ln(3(2x)) = 2$

$\ln(6x) = 2$

$\frac{e^2}{6} = \frac{6x}{6}$

$x = \frac{e^2}{6} \approx \boxed{1.23}$

- $\frac{\cancel{2}\ln(2x^2)}{\cancel{2}} = \frac{1}{2}$

$\ln 2x^2 = \frac{1}{2}$

$\frac{e^{1/2}}{2} = \frac{2x^2}{2}$

$x^2 = \frac{1}{2}e^{1/2}$

$x = \sqrt{\frac{1}{2}e^{1/2}}$

$\boxed{x \approx 0.91}$

- $80e^{-4x} + 16 = 45$

$-16 \qquad -16$

$\frac{80e^{-4x}}{80} = \frac{29}{80}$

$e^{-4x} = 29/80$

$\frac{\ln\frac{29}{80}}{-4} = \frac{-4x}{-4}$

$\boxed{x \approx 0.25}$

## Bacteria Colony

The growth of a certain bacteria colony is modeled by the equation

$$y = y_0 e^{kx}$$

where $x$ is measured in minutes. A scientist began monitoring this colony when there was 20 bacteria present and noticed it took 7 minutes for the colony to increase to 30. How many bacteria were there after 5 minutes?

| x | y |
|---|---|
| 0 | 20 |
| 7 | 30 |
| 5 | ? |

$y = 20e^{kx}$

$y = 20e^{5k}$

$\frac{30}{20} = \frac{20e^{7k}}{20}$

$1.5 = e^{7k}$

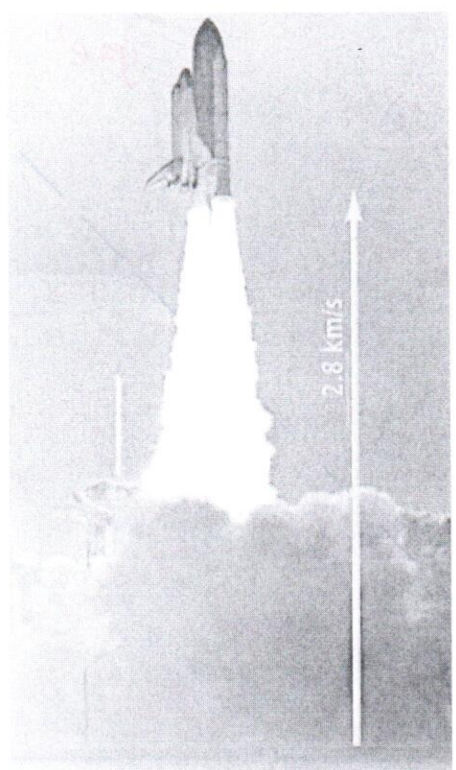
$\frac{\ln 1.5}{7} = \frac{7k}{7}$

$k \approx 0.058$

$y = 20e^{(5 \cdot 0.058)}$

$\boxed{y = 26.73 \text{ bacteria}}$

SPACECRAFT TIME!
(Earn a Stamp for answering this question on your own!)



# When Did the Cougar Die?

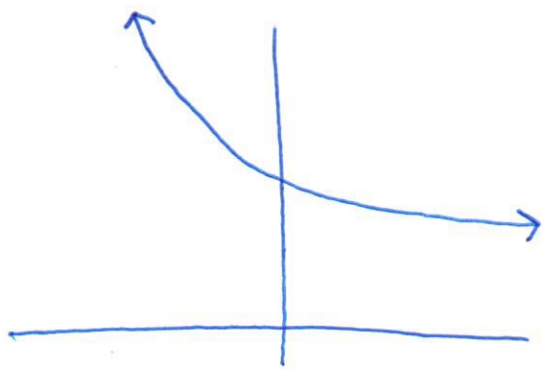
Many states in the western U.S. regard cougars (also known as mountain lion, puma, panther) as game animals with a designated hunting season. Typically, hunting seasons occur in the fall and winter with hunting allowed only during daylight hours (8AM to 6PM). Wildlife Conservation Officers (COs) have the task of verifying that hunters kill cougars only during the specified times. Typically, this is accomplished by requiring successful hunters to visit a check station where legal cougars are tagged and illegal cougars are confiscated with dire consequences for the hapless hunter.

At 10PM, hunters arrived at the checkpoint with a cougar. Upon arrival, officers noted that the air temperature was a mild $68°$ F and the cougar's body temperature was $85°$. At midnight, after questions had been asked and paperwork filed, the cougar's body had further cooled to $74°$.

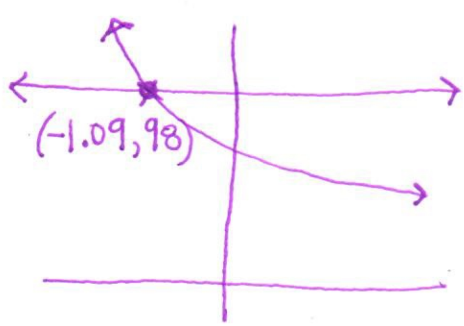
Knowing Newton's law of cooling, the COs were able to determine the time of the cougar's death.

$T = 68° + 17°e^{-0.5207t}$ where t is in hours after 10PM.

Graph the function.



Assuming the cougar's living temperature is $\underset{y}{\underline{98°}}$, determine the cougar's $\underset{x}{\underline{\text{time of death}}}$.




1.09 x 60 mins

65.4 mins

BEFORE 10 PM

$\boxed{\text{8:55 PM Time of death}}$