

Normal Distributions

OBJECTIVES:

- Summarize, represent, and interpret data on a single count or measurement variable
- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages.
- Recognize that there are data sets for which such a procedure is not appropriate.
- Estimate areas under the normal curve.

Collect Data. How many people have you kissed? Please write your **true** number without a name on the post-it note given to you.

VOCABULARY:

Histogram separates the data into intervals of equal width called "Bins" and then counts how many observations fell within each interval.



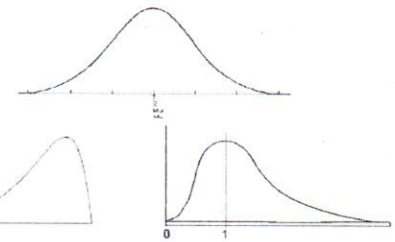
How is the data for our class kisses distributed??

skewed right

Density Curves

Things to know about density curves:

- 1) always on or above x-axis
- 2) area under curve is always = 1. b/c 100% of observations are within the curve.
- 3) since the curve is an approx. of the overall pattern \rightarrow outliers are not seen.



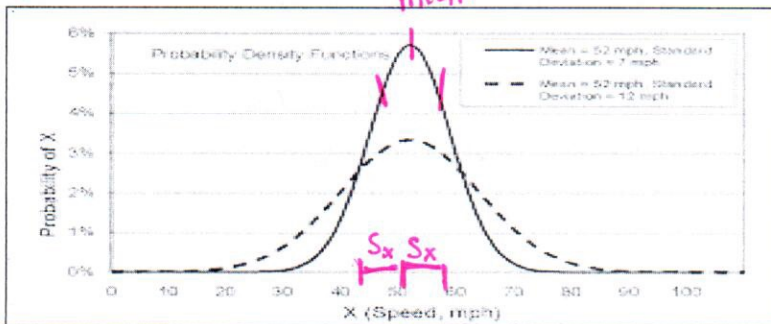
Standard Deviation (S_x):

*** Standard deviation is the typical distance of the values in the data set from the mean

Write this out twice!! _____

The Idea of a Standard deviation (Represents spread of data)

Draw dot plot and then the dot plot re-drawn with distance from the mean averaged out.



The solid taller curve has a standard deviation of 7, the dashed shorter curve has a standard deviation of 12.

They both have the same mean and an area of 1 under the curve

SD by hand	*SD Using a calculator*
$S_x = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}}$ <p>x_1 = first data value \bar{x} = mean of the data set n = number of values in the data set</p> <p>STEPS</p> <ol style="list-style-type: none"> 1. Subtract mean from each pt. 2. square each difference <i>make everything positive</i> 3. Sum up all squared differences 4. Divide by n-1 5. Take the square root 	<p>STATS -> Edit -> L₁ enter in Data</p> <p>STATS -> Calc -> 1-Var Statistics -> L₁ (2nd 1) -> enter</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p>EDIT CALC TESTS 1: Edit 2: SortA() 3: SortD() 4: C1/List 5: SetUpEditor</p> </div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-top: 10px;"> <p>EDIT CALC TESTS 1: 1-Var Stats 2: 2-Var Stats 3: Med-Med 4: LinReg(ax+b) 5: QuadReg 6: CubicReg 7: QuartReg</p> </div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-top: 10px;"> <p>1-Var Stats $\bar{x}=10$ $\Sigma x=130$ $\Sigma x^2=1396$ $S_x=2.828427125$ $\sigma_x=2.717464882$ $n=13$</p> <p><i>std dev</i> (circled) <i>sample</i> (circled) <i>populatio</i> (circled)</p> </div>
<p>Find the standard deviation of the heights of five starters on a basketball team: 67, 72, 76, 76, 84.</p> $S_x = \sqrt{\frac{(67-75)^2 + (72-75)^2 + (76-75)^2 + (76-75)^2 + (84-75)^2}{5-1}} = \sqrt{\frac{41+9+1+1+81}{4}} = \sqrt{39} = 6.245$ <p><i>S_x = 6.245</i></p>	<p>Find the standard deviation of the heights of five starters on a basketball team: 67, 72, 76, 76, 84.</p> $S_x = 6.245$

A Specific type of distribution- **NORMAL DISTRIBUTIONS.**

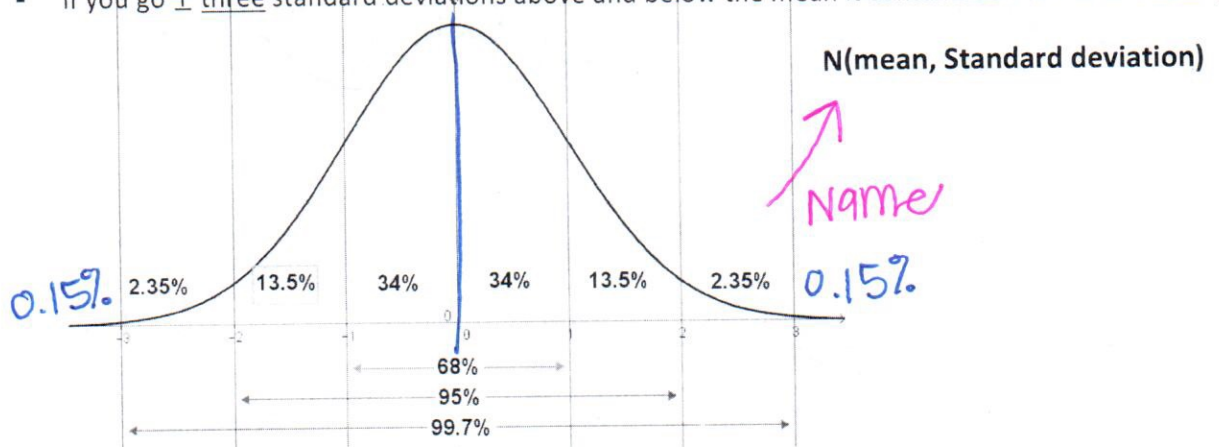
Discovered by multiple mathematicians, but Gauss is generally noted to have made the discovery around 1809.

He notice that many natural phenomenon follow at least an approximately normal distribution and derived the formulas, properties and behaviors of them.

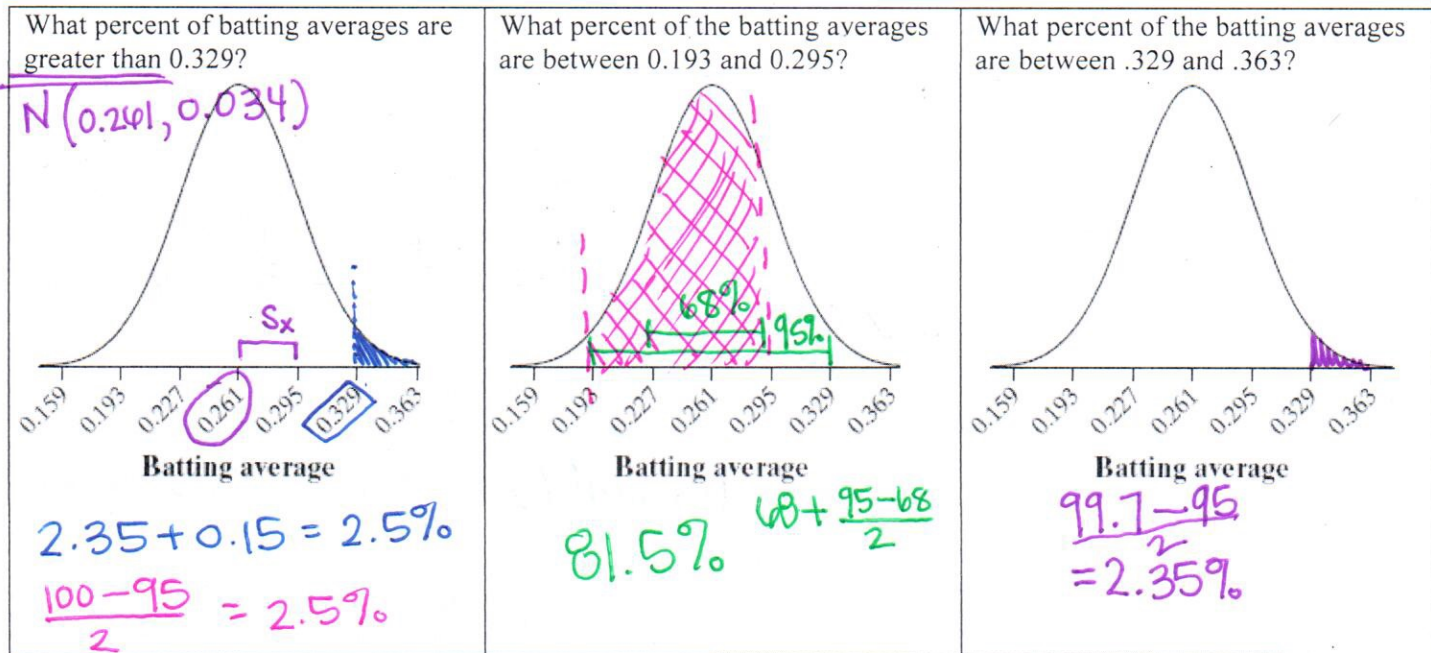
EX. Hair length, height, error in measurements, blood pressure and many more.

- A **Normal Distribution** is a probability distribution that has ALL of the following specific characteristics:

- The distribution is "bell" shaped
- it has one peak (called unimodal)
- it is symmetric with the left half being a mirror image of the right half
- **FOLLOWS THE 68-95-99.7 RULE (EMPIRICAL RULE)**
 - If you go \pm one standard deviation above and below the mean it will contain 68% of the data
 - If you go \pm two standard deviations above and below the mean it will contain 95% of the data
 - If you go \pm three standard deviations above and below the mean it contains 99.7% of the data



Example: Here is a Normal curve for the distribution of batting averages. The mean and the points one, two, and three standard deviations from the mean are labeled.



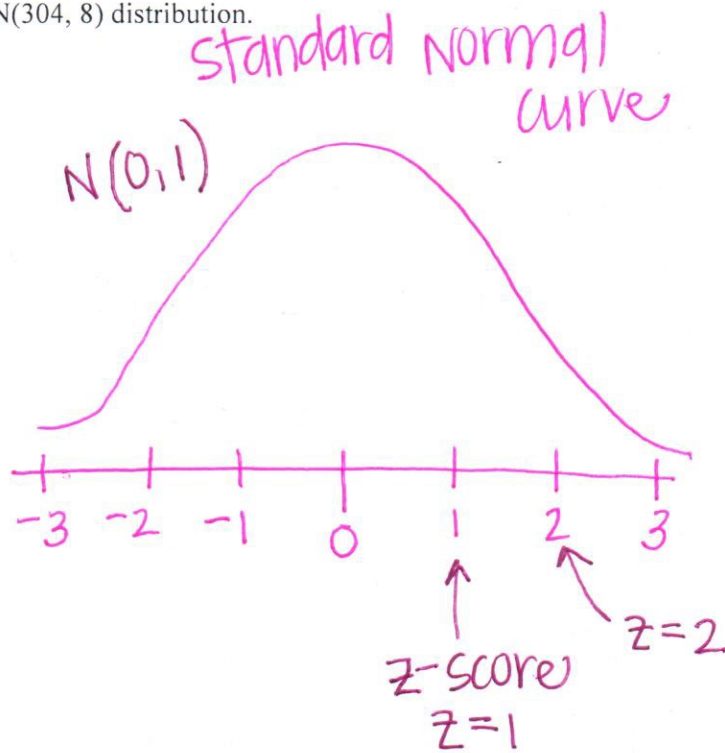
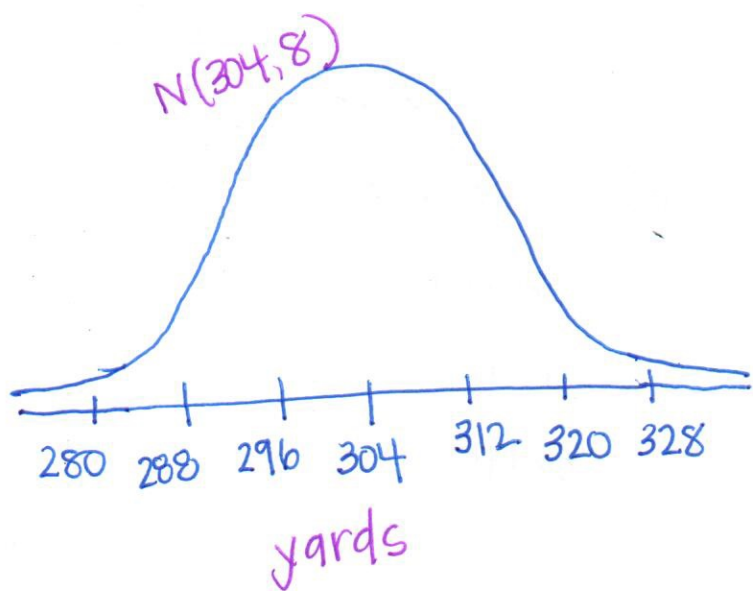
**Normal or Not with Projector

Fitting a normal distribution

Example: Tiger on the Range

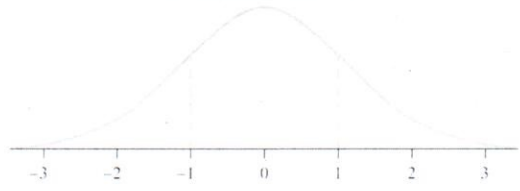
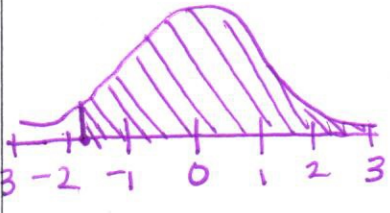
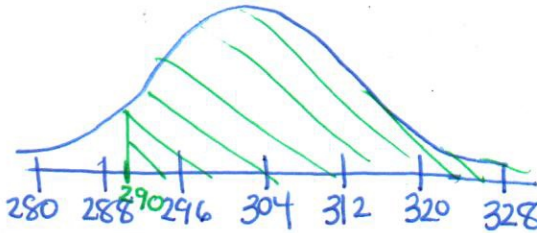
On the driving range, Tiger Woods practices his swing with a particular club by hitting many balls. Suppose that when Tiger uses his driver, the distance the ball travels follows a $N(304, 8)$ distribution.

- Procedure:
1. Draw Curve
 2. Label $N(\text{mean}, \text{standard deviation})$
 3. Mark mean, count by SD's

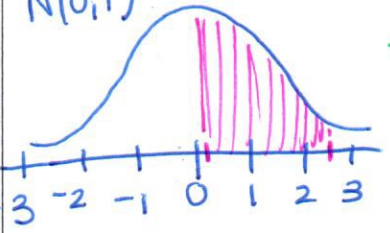


$$x=290$$

What percent of Tiger's drives travel at least 290 yards?

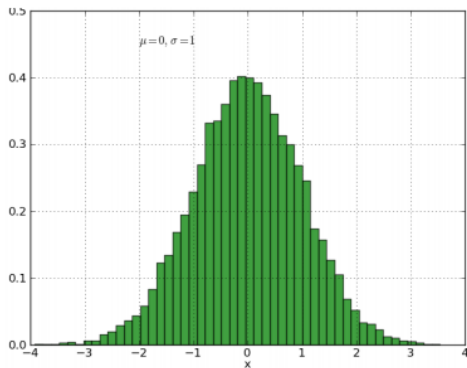
Standardize	Use Calc
 <p>All normal curves can be standardized using the formula:</p> $z = \frac{\text{value} - \text{mean}}{\text{standard deviation}} = \frac{X - \bar{X}}{S_x}$ <p>*A Standard normal curve is the same curve but</p> <ol style="list-style-type: none"> 1. Is centered at 0 2. the scale is in Standard deviations. SD=1 <p>ALWAYS DRAW THE STANDARD CURVE FIRST.</p> $z = \frac{290 - 304}{8} = -1.75$  <p>from table $1 - 0.0401$ $= 0.9599$</p> <p>96% of tiger's drives were ≥ 290 yds.</p>	 <p>2nd Vars (DIST) \rightarrow normcdf</p> <p>Normcdf(Lower Boundary, Upper Boundary, mean, SD)</p> <p>*Use a really big \pm number to represent ∞</p> $\text{normcdf}(290, 9999999, 304, 8)$ <p>0.9599</p> <p>96% of Tiger's drives were ≥ 290 yds.</p>

What percent of Tiger's drives travel between 305 and 325 yards?

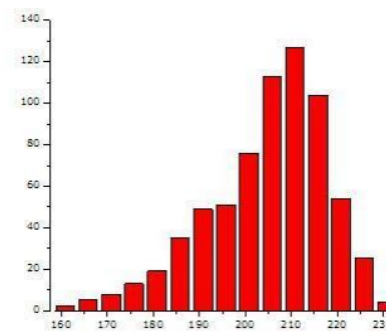
<p>$N(0,1)$</p>  $z_1 = \frac{305 - 304}{8}$ $z_1 = 0.125$ $z_2 = \frac{325 - 304}{8}$ $z_2 = 2.625$ <p>0.5517 0.9957</p> $\frac{0.9957 - 0.5517}{0.444}$ <p>44% of Tiger's drives blw 305 & 325 yds.</p>	$P(305 < x < 325)$ $\text{Normalcdf}(305, 325, 304, 8)$ <p>= 0.4459</p> <p>= 44.6</p> <p>$\approx 45\%$</p>
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What is Normal?

1. This is normal:

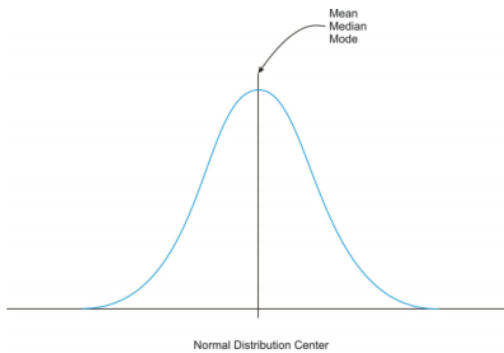


This is not:

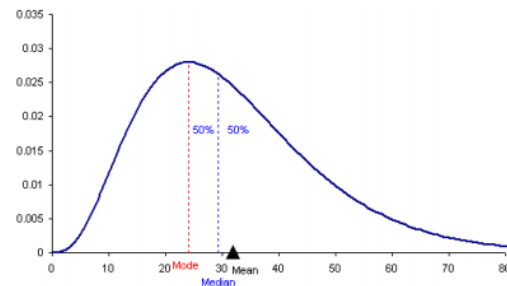


What differences do you see between these distributions?

2. This is normal:

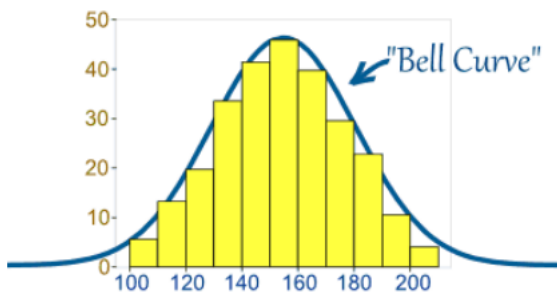


This is not:

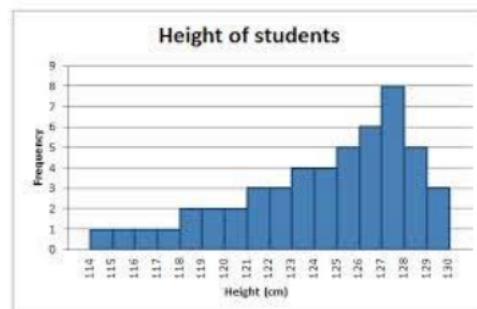


What differences do you see between these distributions?

3. This is normal:

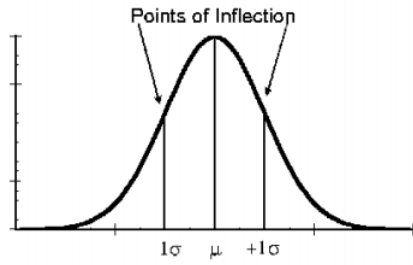


This is not:

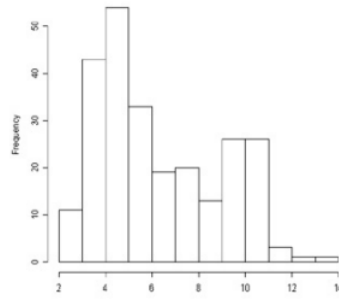


What differences do you see between these distributions?

4. This is normal:

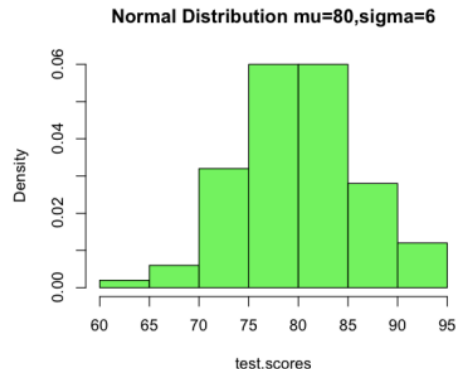


This is not:

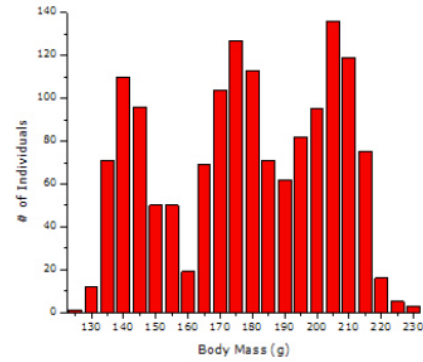


What differences do you see between these distributions?

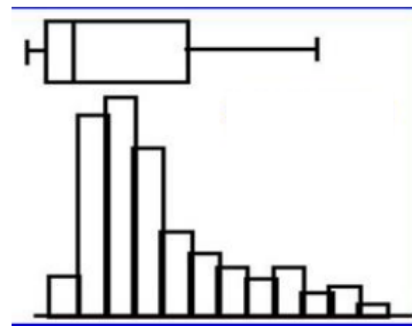
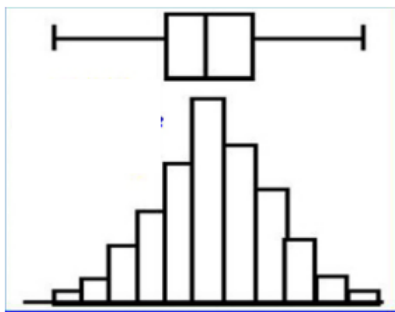
5. This is normal:



This is not:

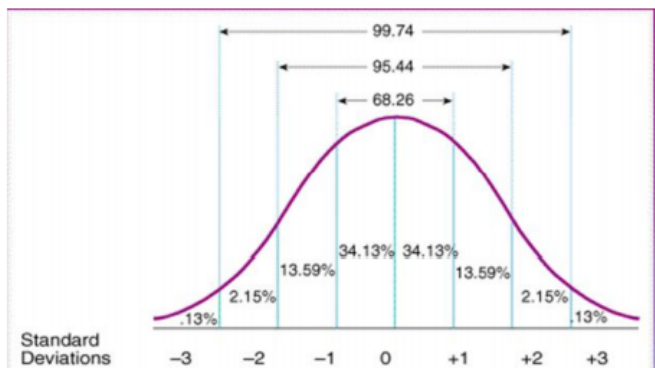


6. This is normal:

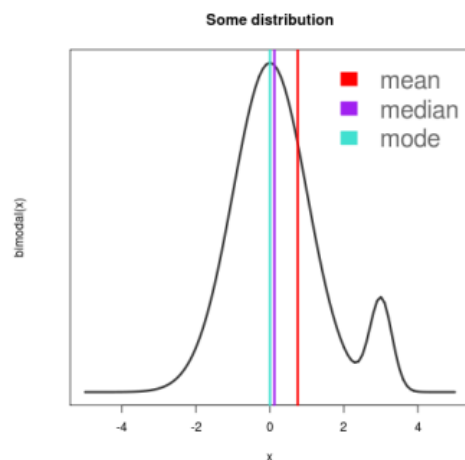


What differences do you see between these distributions?

7. This is normal:



This is not:



What differences do you see between these distributions?