www.khanacademy.org/math/statistics-probability/confidence-intervals-one-sample/introduction-to-confidence-intervals/v/confidence-intervals-and-margin-of-error

Secondary Math 3 Honors

7-5 Margin of error discovery through simulation – NOTES

Objectives:
- Discover and be able to explain the concept of margin of error that comes from random chance when sampling.
- Given the formula for margin of error, be able to construct a confidence interval

Margin of Error for Estimating a Population Mean

Researchers have questioned whether the traditional value of 98.6°F is correct for a typical body temperature for healthy adults. Suppose that you plan to estimate mean body temperature by recording the temperatures of the people in a random sample of 10 healthy adults and calculating the sample mean. How accurate can you expect that estimate to be?

Claim (mean 98.6 degrees and standard deviation 0.7 degrees) $\bar{x}=98.6$ $S_x=0.7$
Researchers took one random sample of 10 healthy adults from this population to test this claim:

97.73	98.76	98.27	99.95	98.47	98.49	98.97	98.68	99.27	99.25

a. What is the mean temperature and standard deviation for this sample?

Stats
- **-edit L_1**

Stats
- -Calc
  - -1VarStat

$\bar{x}=98.784$
$S_x=0.616$

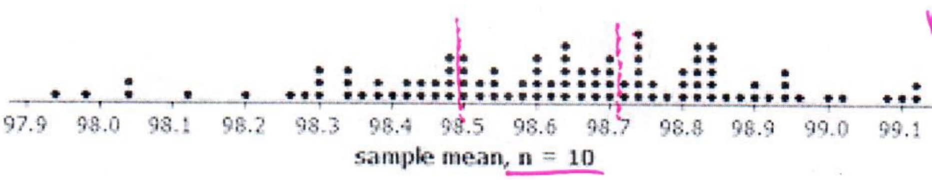
If you were to take a different random sample of size 10, would you expect to get the same value for the sample mean? Explain. NO. Explain in your own words.

Sampling Distribution

Below is a dot plot of the sample mean body temperature for 100 different random samples of size 10 from a population where the mean temperature is 98.6 degrees.

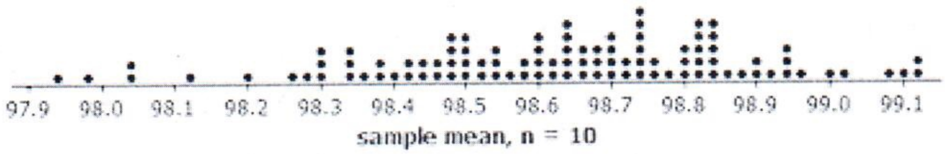
b. How many of the samples had sample means that were greater or equal to 98.5 degrees and less than or equal to 98.7 degrees? 33 samples

Sampling Distribution
$n=10$



c. Based on the dot plot above, if you were to take a different random sample from the population, would you be surprised if you got a sample mean of 98.8 or greater? Explain why or why not.

NO. 28 samples were $\geq 98.8°$.
SO $\frac{28}{100}=28\%$, we would have 28% probability of observing a sample mean of 98.8 by chance. This is not unusual.



d. Which of the following statements is appropriate based on the dot plot of sample means above?

Statement 1:Most random samples of size 10 from the population would result in a sample mean that is within 0.1 degrees of the value of the population mean (98.6).

Statement 2:Most random samples of size 10 from the population would result in a sample mean that is within 0.3 degrees of the value of the population mean (98.6).

Statement 3:Most random samples of size 10 from the population would result in a sample mean that is within 0.5 degrees of the value of the population mean (98.6).

only 6 samples are not w/in 0.5 degrees of mean.

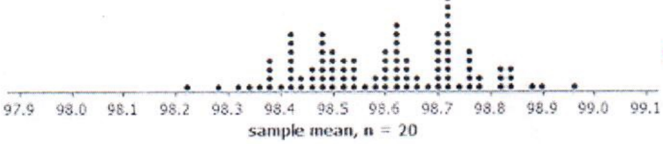
The **margin of error** is the maximum likely difference between the estimate and the actual value of the population mean for a given sample size. (or the maximum expected difference between the true population parameter and a sample estimate of that parameter.

e. Explain why 0.45 degrees would be a reasonable estimate of the margin of error for a sample size of 10. any sample we might get by chance is within 0.45 of the population mean.

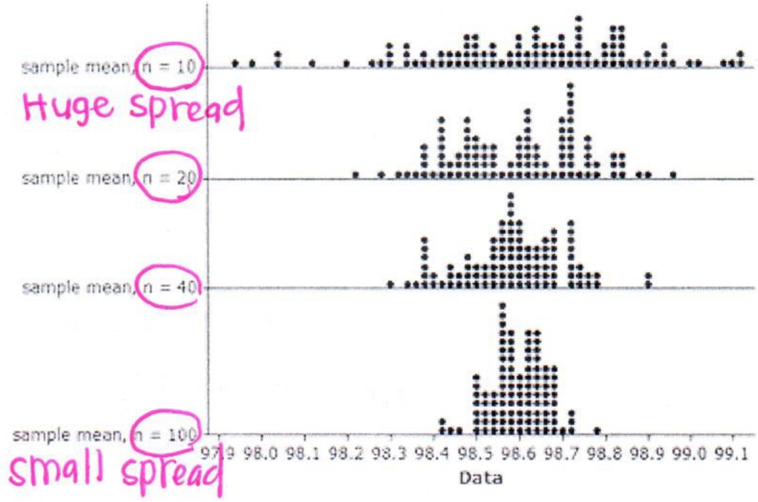
f. If you were to increase your sample size to 20 to estimate the population mean, how would that affect the margin of error? The larger the sample size, the more accurate temperature readings you get and therefore the margin of error is smaller. You have better info with a larger sample.

g. Below is a dot plot of the sample mean temperature for 100 different random samples of size 20 from a population with an actual mean temperature of 98.6 degrees. Explain how this dot plot supports your answer in part (f). Make sure to provide a new reasonable estimate of the margin of error.

- Dots are closer to 98.6. Less spread out (variance)
- Most are within 0.3 degrees. (margin of error)



Below is a comparative dot plot that shows sample means for 100 random samples for each of the sample sizes 10, 20, 40, and 100.



Huge spread

small spread

h. How is the margin of error related to sample size?

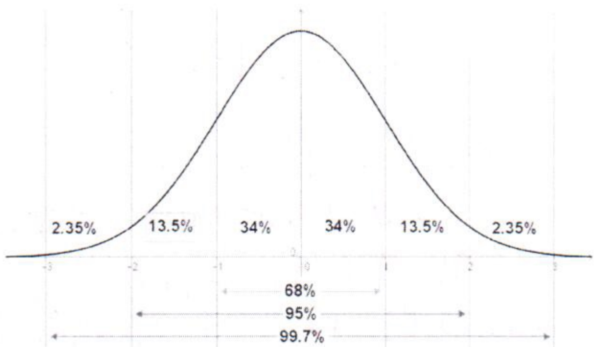
- small margin of error for larger sample size.
- It gets smaller as sample size increases.

Putting this concept to use

In actuality, we don't take many random samples from a population because, in many cases, we already know what the distribution of the samples should look like.

Because we are working with sample means, in most cases, we know the sampling distribution will be approximately normally distributed.

Because of this, we can estimate the margin of error from a single sample for any percentage of confidence we would like.



An estimate of the margin of error based on a single random sample can be obtained by evaluating the following expression

$$\text{Estimated margin of error} = 1.96\frac{s_x}{\sqrt{n}}$$

S_x = standard deviation
n = sample size

i. Using the sample at the beginning of this activity, what is the estimated margin of error?

$S_x = 0.616$
$n = 10$

$ME = 1.96\left(\frac{0.616}{\sqrt{10}}\right) = 0.382$

j. Construct a 95% confidence interval

$\bar{x} \pm ME$

$\bar{x} = 98.784$
$ME = 0.382$

$$\begin{array}{r} 98.784 \\ -\ 0.382 \\ \hline 98.402 \end{array} \qquad \begin{array}{r} 98.784 \\ +\ 0.382 \\ \hline 99.166 \end{array}$$

$\boxed{(98.402, 99.166)}$
95% C.I.

k. Interpret the confidence Interval

We are 95% confident that the true mean body temperature in adults, is between 98.402 and 99.166 degrees.

l. Does this confidence interval support the claim of a mean body temperature of 98.6 degrees?

m. Yes, because 98.6° is in our confidence interval.

n. Practice:

Suppose that a random sample of 50 healthy adults resulted in a sample mean body temperature of $\bar{x}$ = 98.2 degrees and a sample standard deviation of s_x = 0.65 degrees.

Would you consider this evidence that the actual mean temperature for healthy adults is in fact less than 98.6 degrees? (Hint: what is the estimated margin of error?)

$n = 50$

$\bar{x} = 98.2$

$S_x = 0.65$

$$ME = 1.96\left(\frac{0.65}{\sqrt{50}}\right)$$

$= 0.180$

$98.2 - 0.18 = 98.02$

$98.2 + 0.18 = 98.38$

$(98.02, 98.38)$

98.6° is NOT in the 95% C.I. therefore we have significant statistical evidence that the true mean body temperature in adults is NOT 98.6°, but acutually less than.

o. Statistically literate and interpreting statistics with margin of error.

In recent elections, we have seen something like this from news reporters.

NEWS reporter: From the exit polls, we are showing that Billy is going to take the win. 52.5% of the votes are for Billy with a margin of error of .03.

Can this news reporter make the claim that it looks like Billy is going to win?

$ME = 0.03 = 3\%$

$\bar{x} = 52.5$

$52.5 - 3 = 49.5\%$ $\quad$ $52.5 + 3 = 55.5\%$

Because 49.5% is the low end of our confidence interval, it is still possible that Billy could get less than 50% of the votes and could in fact lose the election.

We are 95% confident Billy could get between 49.5% and 55.% of the votes.