Secondary 3 Honors: Unit 8: Trigonometry Name: ____________________

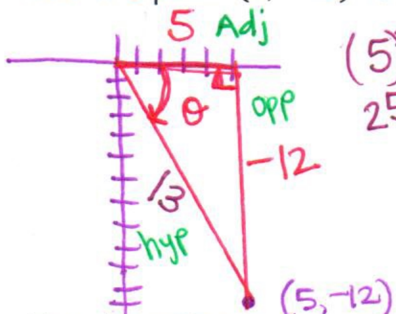
Lesson 8-3: Extending Trig Functions Beyond Right Triangles Notes

Today we will learn:

- That the trig functions can be represented as a circular periodic function.
- What a reference angle is and how to find one.
- How to use reference angles and the unit circle to find exact trig values.

Using a point to evaluate the six trig functions of an angle.

- Use the point $(5, -12)$ to evaluate the six trigonometric functions of the angle.



$(5)^2 + (-12)^2 = c^2$

$25 + 144 = c^2$

$169 = c^2$

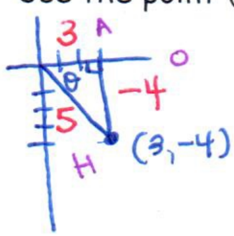
$c = 13$

$\sin\theta = \frac{-12}{13}$ $\csc\theta = \frac{13}{-12}$

$\cos\theta = \frac{5}{13}$ $\sec\theta = \frac{13}{5}$

$\tan\theta = \frac{-12}{5}$ $\cot\theta = \frac{5}{-12}$

- Use the point $(3, -4)$ to evaluate the six trigonometric functions of the angle.



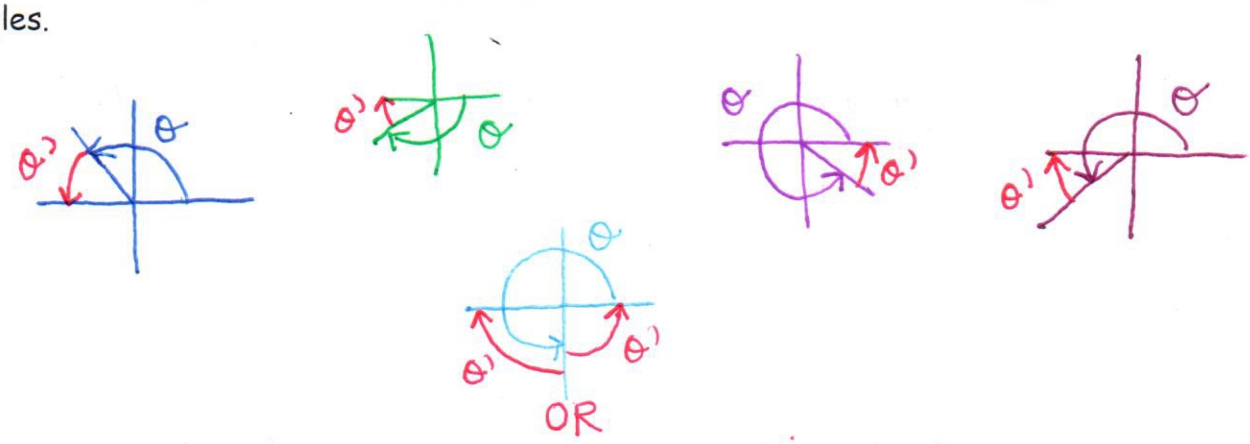
$\sin\theta = \frac{-4}{5}$ $\csc\theta = \frac{5}{-4}$

$\cos\theta = 3/5$ $\sec\theta = 5/3$

$\tan\theta = -4/3$ $\cot = \frac{3}{-4}$

REFERENCE ANGLES: are always POSITIVE and $\leq 90°$!!!!

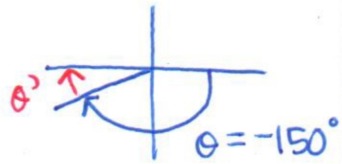
The values of trig functions of angles greater than $90°$ can be found using reference angles.



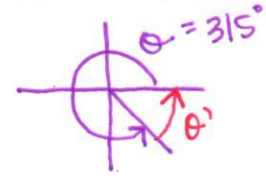
*We always draw from our angle back to the x-axis using the shortest path.

Practice finding reference angles: (in degrees)

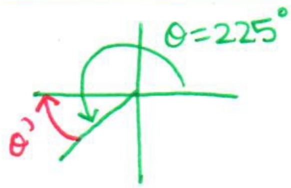
$\theta = -150°$
$\theta' =$ 30°



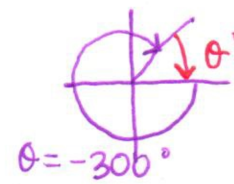
$\theta = 315°$
$\theta' =$ 45°



$\theta = 225°$
$\theta' =$ 45°

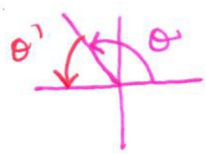


$\theta = -300°$
$\theta' =$ 60°

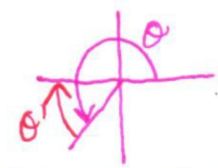


Your turn:

$\theta = 120°$
$\theta' =$ 60°

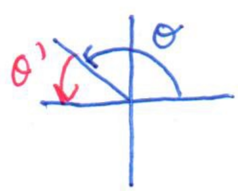


$\theta = 240°$
$\theta' =$ 60°

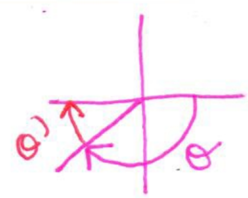


Practice finding reference angles: (in radians)

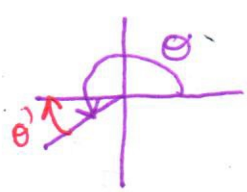
$\theta = \frac{3\pi}{4}$
$\theta' = \frac{\pi}{4}$



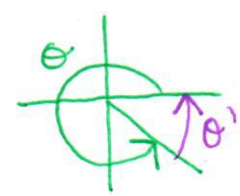
$\theta = \frac{-2\pi}{3}$
$\theta' = \frac{\pi}{3}$



$\theta = \frac{7\pi}{6}$
$\theta' = \frac{\pi}{6}$

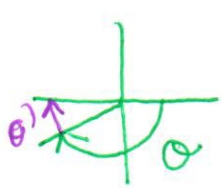


$\theta = \frac{7\pi}{4}$
$\theta' = \frac{\pi}{4}$

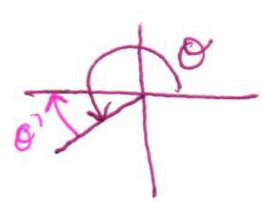


Your turn:

$\theta = \frac{-5\pi}{6}$
$\theta' = \frac{\pi}{6}$



$\theta = \frac{4\pi}{3}$
$\theta' = \frac{\pi}{3}$



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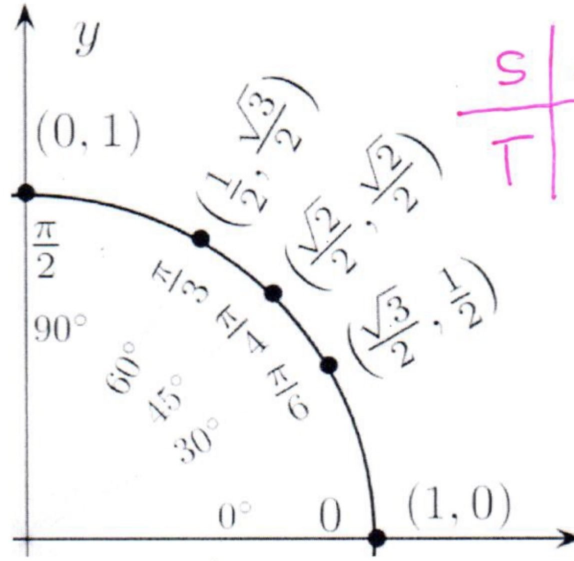
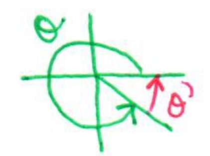
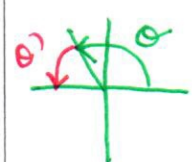
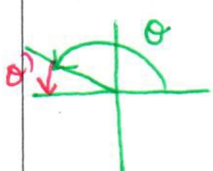
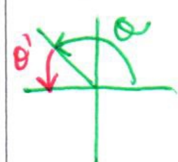
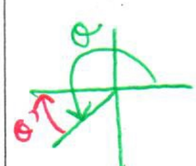
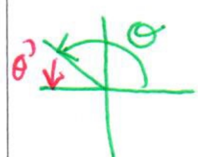
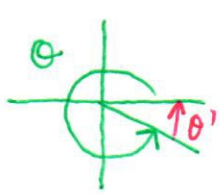
Why do we care about reference angles???

They help us find the exact values of Trig functions BEYOND right $\triangle$'s. Also, we only need to memorize QI of the unit circle.



Evaluating the function without using a calculator or your unit circle…

$\sin 315°$  $\theta = 315°$ $\theta' = 45°$ QIV $(-)$ $\sin 45° = \frac{\sqrt{2}}{2}$ $\sin 315° = -\frac{\sqrt{2}}{2}$	$\sin 120°$  $\theta = 120°$ $\theta' = 60°$ QII $(+)$ $\sin 60° = \frac{\sqrt{3}}{2}$ $\sin 120° = \frac{\sqrt{3}}{2}$
$\cos 150°$  $\theta = 150°$ $\theta' = 30°$ QII $(-)$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\cos 150° = -\frac{\sqrt{3}}{2}$	$\tan \frac{3\pi}{4}$  $\theta = \frac{3\pi}{4}$ $\theta' = \frac{\pi}{4}$ QII $(-)$ $\tan\frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$ $\tan\frac{3\pi}{4} = -1$
$\frac{1}{\cos 240°}$ $\sec 240°$  $\theta = 240°$ $\theta' = 60°$ QIII $(-)$ $\cos 60° = \frac{1}{2}$ $\sec 60° = 2$ $\sec 240° = -2$	$\csc \frac{5\pi}{6}$  $\theta = 5\pi/6$ $\theta' = \pi/6$ QII $(+)$ $\sin \pi/6 = \frac{1}{2}$ $\csc \pi/6 = 2$ $\csc 5\pi/6 = 2$
$\cot \frac{11\pi}{6}$  $\theta = \frac{11\pi}{6}$ $\theta' = \frac{\pi}{6}$ QIV $(-)$ $\cot\frac{\pi}{6} = \frac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}} = \frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}} = \sqrt{3}$ $\cot\frac{11\pi}{6} = -\sqrt{3}$	