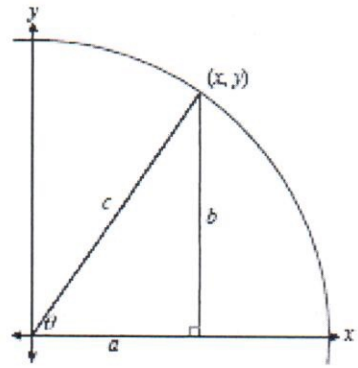
Secondary Math 3 Honors Name:____________________

Unit 9: Trigonometric Identities:

Day 1: Fundamental Trigonometric Identities

An **identity** is an equation that is true for its domain.

In the figure below, the point (x, y) is a point on a circle with radius c. By working with some of the relationships that exist between the quantities in this figure, you will arrive at the first of the Pythagorean Identities.



1. Use the Pythagorean Theorem to write an equation that relates $a, b, and\ c$. $a^2+b^2=c^2$

2. What ratio is equal to $\cos\theta$? $\cos\theta = \frac{a}{c}$

3. What ratio is equal to $\sin\theta$? $\sin\theta = \frac{b}{c}$

4. Using substitution and simplification, combine the three equations from parts 1-3 into a single equation that is only in terms of θ. This equation is the first of the three Pythagorean Identities.

$c\cdot\cos\theta = \frac{a}{\cancel{c}}\cdot\cancel{c}$ $\quad c\cdot\sin\theta = \frac{b}{\cancel{c}}\cdot\cancel{c}$

$a = c\cos\theta$ $\quad b = c\sin\theta$

$a^2+b^2=c^2$

$(c\cos\theta)^2+(c\sin\theta)^2=c^2$

$\frac{\cancel{c^2}\cos^2\theta}{\cancel{c^2}}+\frac{\cancel{c^2}\sin^2\theta}{\cancel{c^2}}=\frac{\cancel{c^2}}{\cancel{c^2}}$

$\boxed{\cos^2\theta+\sin^2\theta=1}$

The other two Pythagorean identities can be derived directly from the first. In order to make these simplifications, you will need to recall the definitions of the other four trigonometric functions.

$\tan x = \frac{\sin x}{\cos x}$ $\qquad \cot x = \frac{\cos x}{\sin x}$ $\qquad \sec x = \frac{1}{\cos x}$ $\qquad \csc x = \frac{1}{\sin x}$

1. Divide both sides of the first Pythagorean identity by $\cos^2\theta$ and simplify.

$\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$

$\boxed{\tan^2\theta+1=\sec^2\theta}$

2. Divide both sides of the first Pythagorean identity by $\sin^2\theta$ and simplify.

$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$

$\boxed{1+\cot^2\theta=\csc^2\theta}$

Fundamental Trigonometric Identities

Reciprocal Identities	$\csc x = \dfrac{1}{\sin x}$	$\sec x = \dfrac{1}{\cos x}$	$\cot x = \dfrac{1}{\tan x}$
Ratio Identities	$\tan x = \dfrac{\sin x}{\cos x}$		$\cot x = \dfrac{\cos x}{\sin x}$
Pythagorean Identities	$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$

Verification of Trigonometric Identities

To verify an identity, we show that one side of the identity can be rewritten in an equivalent form that is identical to the other side. There is no one method that can be used to verify every identity; however, the following guidelines should prove useful.

Guideline for Verifying Trigonometric Identities
▪ If one side of the identity is more complex than the other, then it is generally the best to try first to simplify the more complex side until it become identical to the other side.
▪ Perform indicated operations such as adding fractions or squaring a binomial. Also be aware of any factorization that may help you to achieve your goal of producing the expression on the other side.
▪ Make use of previously established identities that enable you to rewrite one side of the identity in an equivalent form.
▪ Rewrite one side of the identity so that it involves only sines and/or cosines.
▪ Rewrite one side of the identity in terms of a single trigonometric function.
▪ Multiplying both the numerator and denominator of a fraction by the same factor (such as the conjugate of the denominator or the conjugate of the numerator) may get you closer to your goal.
▪ Keep your goal in mind. Does it involve products, quotients, sums, radicals, or powers? Knowing exactly what your goal is may provide the insight you need to verify the identity.

The 4 "S's" and a "C" Simplifying/Verifying Trig. Identities

1) **Sine**- change to equivalent forms of sine or cosine

2) **Single** fraction- make the expression into a single fraction

3) **Square** terms- replace terms with their Pythagorean identity equivalent

4) **Simplify**- divide out, factor, combine like terms, etc.

Conjugate- multiply by the conjugate

Example 1: Change to Sines and Cosines to Verify an Identity

Verify the identity $\boxed{\sin x \cot x \sec x} = 1$

$\sin x \cot x \sec x$	complex side
$\frac{\sin x}{1}\left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x}\right)$	sines/cosines
$\frac{\cancel{\sin x}\,\cancel{\cos x}}{\cancel{\sin x}\,\cancel{\cos x}}$	multiply
	simplify
1	■

Example 2: Use a Pythagorean Identity to Verify an Identity

Verify the identity $1 - 2\sin^2 x = \boxed{2\cos^2 x - 1}$

$2\cos^2 x - 1$	complex side
$2[1-\sin^2 x] - 1$	pyth. identity
$2 - 2\sin^2 x - 1$	Distribute
$1 - 2\sin^2 x$	combine like terms
	■

Example 3: Factor to Verify an Identity

Verify the identity $\boxed{\csc^2 x - \cos^2 x \csc^2 x} = 1$

$\csc^2 x - \cos^2 x \csc^2 x$	complex side
$\csc^2 x(1-\cos^2 x)$	factor
$\csc^2 x \sin^2 x$	pyth. identity
$\frac{1}{\sin^2 x}\sin^2 x$	reciprocal identity
$\frac{\sin^2 x}{\sin^2 x}$	Multiply
1	simplify ■

Example 4: Multiply by a Conjugate to Verify an Identity

Verify the identity $\frac{\sin x}{1+\cos x} = \boxed{\frac{1-\cos x}{\sin x}}$

$\frac{1-\cos x}{\sin x}$	complex side
$\frac{(1+\cos x)}{(1+\cos x)} \cdot \frac{(1-\cos x)}{(\sin x)}$	conjugate
$\frac{1-\cos^2 x}{(1+\cos x)(\sin x)}$	Multiply
$\frac{\sin^{\cancel{2}} x}{(1+\cos x)(\cancel{\sin x})}$	pyth. identity cancel
$\frac{\sin x}{1+\cos x}$	

Example 5: Change to Sines and Cosines to Verify an Identity

Verify the identity $\boxed{\frac{\sin x+\tan x}{1+\cos x}} = \tan x$

$\frac{\sin x + \tan x}{1+\cos x}$	complex side
$\frac{\cos x}{\cos x} \frac{\left(\frac{\sin x}{1} + \frac{\sin x}{\cos x}\right)}{(1+\cos x)}$	sines/cosines
$\frac{\cos x \sin x + \sin x}{\cos x(1+\cos x)}$	get rid of fractions
$\frac{\sin x\,\cancel{(\cos x+1)}}{\cos x\,\cancel{(1+\cos x)}}$	Factor cancel
$\tan x$	ratio identity