Secondary 3 Honors

Name:____________________________

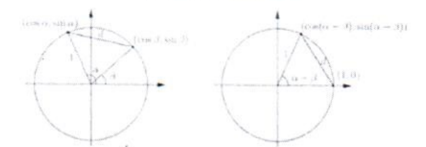
9-3: Sum and Difference Identities Notes

Each Fundamental Identity involved only one variable. We now consider identities that involve a trigonometric function of the sum or difference of two variables.

Sum and Difference Identities
$\cos(\alpha+\beta) =$ $\cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\cos(\alpha-\beta) =$ $\cos\alpha\cos\beta + \sin\alpha\sin\beta$
$\sin(\alpha+\beta) =$ $\sin\alpha\cos\beta + \sin\beta\cos\alpha$
$\sin(\alpha-\beta) =$ $\sin\alpha\cos\beta - \sin\beta\cos\alpha$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

*We could derive the identity for $\cos(\alpha+\beta)$ by drawing a unit circle and two different angles and then using the distance formula.



Our goals today are to use these identities to....

- Evaluate Trigonometric Expressions
- Simplify Trigonometric Expressions
- Verify an Identity

Example 1: Evaluate a Trigonometric Expression

Use an identity to find the *exact* value of $\cos(75°)$

$\cos(\underset{\alpha}{30°}+\underset{\beta}{45°})$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \cos 30°\cos 45° - \sin 30°\sin 45°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

Use an identity to find the *exact* value of $\sin(315°)$

$\sin(\underset{\alpha}{360°}-\underset{\beta}{45°})$

$= \sin 360°\cos 45° - \sin 45°\cos 360°$

$\cancel{(0)\left(\frac{\sqrt{2}}{2}\right)} - \left(\frac{\sqrt{2}}{2}\right)(1)$

$\boxed{\frac{-\sqrt{2}}{2}}$

Use an identity to find the *exact* value of $\cos\left(\frac{\pi}{4}-\frac{\pi}{3}\right)$

$\alpha \quad \beta$

$\cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

Use an identity to find the *exact* value of $\sin\left(\frac{\pi}{6}+\frac{\pi}{2}\right)$

$\alpha \quad \beta$

$\sin\frac{\pi}{6}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\cos\frac{\pi}{6}$

$\cancel{\left(\frac{1}{2}\right)(0)} + (1)\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\frac{\sqrt{3}}{2}}$

Example 2: Simplify Trigonometric Expressions

Write each expression in terms of a single trigonometric function: $\sin \underset{\alpha}{5x} \cos \underset{\beta}{3x} - \cos \underset{\alpha}{5x} \sin \underset{\beta}{3x}$

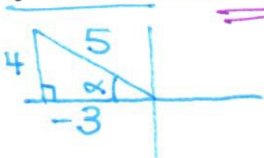
$$\sin(5x - 3x) = \boxed{\sin 2x}$$

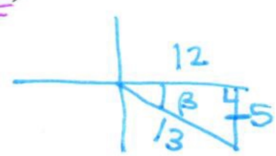
Write each expression in terms of a single trigonometric function: $\cos \underset{\alpha}{x} \cos \underset{\beta}{5x} - \sin \underset{\alpha}{x} \sin \underset{\beta}{5x}$

$$\cos(x + 5x) = \boxed{\cos 6x}$$

Example 3: Evaluate a Trigonometric Expression

Given $\tan \alpha = -\frac{4}{3}$ in Quadrant II and $\tan \beta = -\frac{5}{12}$ in Quadrant IV, find $\sin(\alpha + \beta)$.





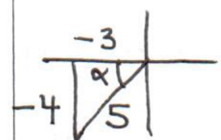
$\sin(\alpha + \beta)$

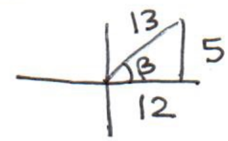
$\sin\alpha \cos\beta + \sin\beta \cos\alpha$

$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right)$

$= \frac{48}{65} + \frac{15}{65} = \boxed{\frac{63}{65}}$

Given $\cos \alpha = -\frac{3}{5}$ in Quadrant III and $\sin \beta = \frac{5}{13}$ in Quadrant I, find $\cos(\alpha - \beta)$.





$\cos(\alpha - \beta)$

$\cos\alpha \cos\beta + \sin\alpha \sin\beta$

$\left(-\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(-\frac{4}{5}\right)\left(\frac{5}{13}\right)$

$-\frac{36}{65} - \frac{20}{65} = \boxed{-\frac{56}{65}}$

Example 4: Verify an Identity

Verify the identity $\boxed{\cos(\pi - \theta)} = -\cos\theta$

complex

$\cos(\underset{}{\overset{\alpha}{\pi}} - \overset{\beta}{\theta})$ — diff. identity

$\cos\pi \cos\theta + \sin\pi \sin\theta$

$(-1)\cos\theta + \cancel{(0)\sin\theta}$ — unit circle

$-\cos\theta$

Verify the identity $\boxed{\frac{\cos 4x}{\sin x} - \frac{\sin 4x}{\cos x}} = \frac{\cos 5x}{\sin x \cos x}$

complex

$\frac{\cos x}{\cos x} \cdot \frac{\cos 4x}{\sin x} - \frac{\sin 4x}{\cos x} \cdot \frac{\sin x}{\sin x}$ — common denom.

$\frac{\cos x \cos 4x}{\sin x \cos x} - \frac{\sin 4x \sin x}{\sin x \cos x}$ — combine

$\frac{\overset{\alpha}{\cos x} \overset{\beta}{\cos 4x} - \overset{\beta}{\sin 4x} \overset{\alpha}{\sin x}}{\sin x \cos x}$ — sum/diff identity

$\frac{\cos(x + 4x)}{\sin x \cos x}$

$\frac{\cos 5x}{\sin x \cos x}$