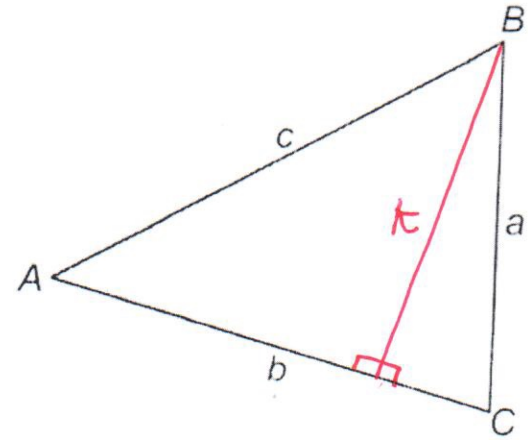
Name: ____ Period: __________

9-4 Notes: Law of Sines

Notes

Consider oblique triangle ΔABC shown to the right.



1. Sketch an altitude from vertex B.
2. Label the altitude k.
3. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles and $\angle C$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $sinA$ and one involving $sinC$.

   $sinA = \frac{k}{c}$ $\qquad$ $sinC = \frac{k}{a}$

4. Notice that each of the equations in Question 3 involves k. Why does this happen?

   one of the sides opposite side to our ∠'s.

   k made the right △'s.

5. Solve each equation above for k.

   $k = c\sin A$ $\qquad$ $k = a\sin C$

6. Since both equations in Question 5 are equal to k, they can be set equal to each other. Why is this possible? Set the equations equal to each other to form a new equation.

   substitution property
   k is the same

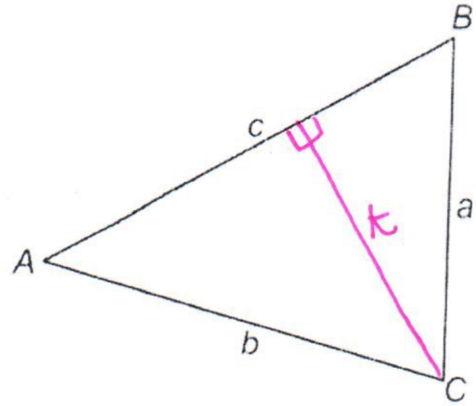
   $c\sin A = a\sin C$

7. Notice that the equation in Question 6 no longer involves k. Why not? Write an equation equivalent to the equation in Question 6, regrouping a with $sinA$ and c with $sinC$.

   we added in k, go back to original △.

   $\frac{\sin A}{a} = \frac{\sin C}{c}$

Again, consider oblique ΔABC.



8. This time, sketch an altitude from vertex C.
9. Label the altitude k.
10. This new altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles and $\angle B$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin A$ and one involving $sinB$.

$sinA = \frac{k}{b}$ $\qquad$ $sinB = \frac{k}{a}$

11. Notice that each of the equations in Question 10 involved k. Why does this happen? Solve each equation for k.

$k = b\sin A$ $\qquad$ $k = a\sin B$ $\qquad$ k is the opposite side for both.

12. Since both equations in Question 11 are equal to k, they can be set equal to each other. Why is this possible? Set the equations equal to each other to form a new equation.

$b\sin A = a\sin B$

13. Notice that the equation in Question 12 no longer involves k. Why not? Write an equation equivalent to the equation in Question 12, regrouping a with $\sin A$ and b with $\sin B$.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

14. Use the equations in Question 7 and Question 13 to write a third equation involving b, c, $\sin B$ and $\sin C$.

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

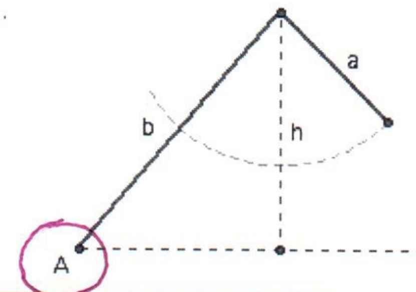
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

Together, the equations in Questions 7, 13, and 14 form the *Law of Sines*. The Law of Sines is important because it can be used to solve problems involving both right AND non-right triangles, because it involves only the sides and angles of a triangle.

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Let's ~~consider~~ consider when we have two consecutive sides, and then angle NOT in between them (aka SSA).

How many different triangles are possible? Follow along with the demonstration.



Angle A	Relationship between opp a and c adj	Relationship between opp a and h height	Number of possible triangles
Acute	$a < b$	$a < h$	NONE
Acute	$a < b$	$a = h$	1 right △
Acute	$a < b$	$h < a$	2 triangles ★
Acute	$a > b$	$a > h$	1 triangle
Obtuse	$a < b \text{ or } a = b$		NONE
Obtuse	$a > b$		1 triangle

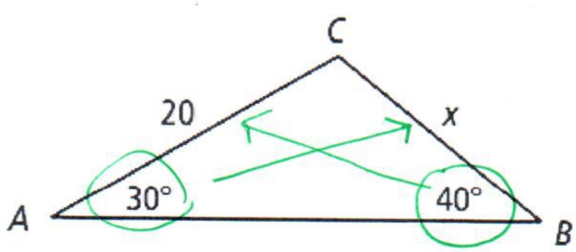
2 △'s

The only time that we have to worry about the ambiguous case is when:

acute angle

height < opposite side   opp side < adj side

Examples: Use the Law of Sines. Find the measure of x to the nearest tenth.

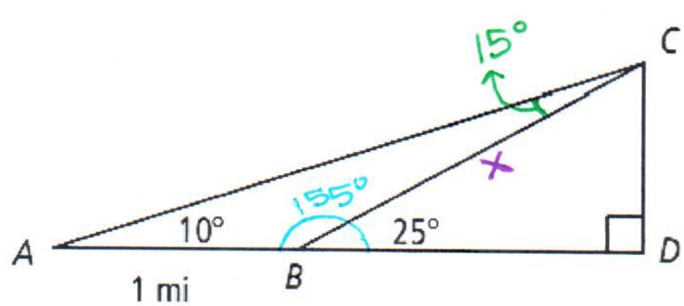


$$\frac{\sin 30°}{x} = \frac{\sin 40°}{20}$$

$$\frac{20\sin 30°}{\sin 40°} = \frac{x\cancel{\sin 40°}}{\cancel{\sin 40°}}$$

$$\boxed{x = 15.6}$$

A surveyor measure the angle to the top of a hill from two different points. The angles she measures and the distance between the valley points are shown in the diagram at the right. What is the distance from point B and the top of the hill? Round your answer to the nearest hundredth of a mile.



$$\frac{\sin 10°}{x} = \frac{\sin 15°}{1}$$

$$\frac{\sin 10°}{\sin 15°} = \frac{x\cancel{\sin 15°}}{\cancel{\sin 15°}}$$

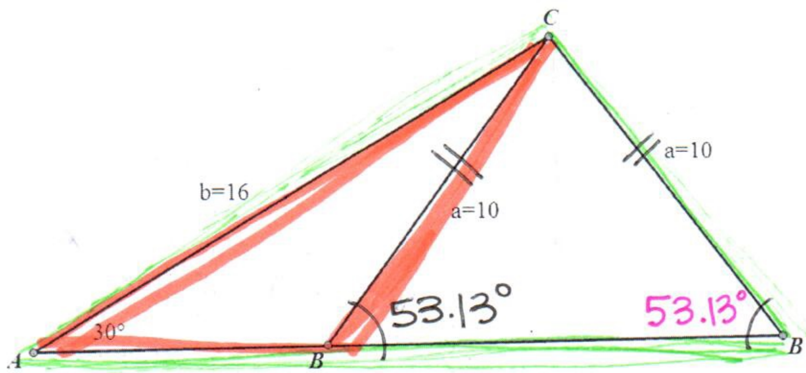
$$\boxed{x = 0.67 \text{ miles}}$$

Handling the ambiguous case: $a > h$

In $\Delta ABC, a = 10, b = 16, m\angle A = 30°$. $a < b$

acute

Find the missing sides and angles of all possible triangles.



Green $\triangle$

$\boxed{\angle B}$ $\frac{\sin 30°}{10} = \frac{\sin B}{16}$

$\frac{16\sin 30°}{10} = \frac{\cancel{10}\sin B}{\cancel{10}}$

$\sin^{-1}(\sin B) = (0.8)$ $\sin^{-1}$

$B = \sin^{-1}(0.8)$

$\boxed{B = 53.13°}$

$\boxed{\angle C}$ $180 - 30 - 53.13$

$\boxed{\angle C = 96.87°}$

$\boxed{\text{side } c}$

$\frac{\sin 30°}{10} = \frac{\sin 96.87°}{c}$

$\frac{c\cancel{\sin 30°}}{\cancel{\sin 30°}} = \frac{10\sin 96.87°}{\sin 30°}$

$\boxed{c = 19.86}$

Orange $\triangle$

$\boxed{\angle B}$ $180 - 53.13°$

$\boxed{\angle B = 126.87°}$

$\boxed{\angle C}$ $180 - 30 - 126.87$

$\boxed{\angle C = 23.13°}$

$\boxed{\text{side } c}$

$\frac{\sin 30°}{10} = \frac{\sin 23.13°}{c}$

$\frac{c\cancel{\sin 30°}}{\cancel{\sin 30°}} = \frac{10\sin 23.13°}{\sin 30°}$

$\boxed{c = 7.86}$