Name: ______________________________ Period: __________

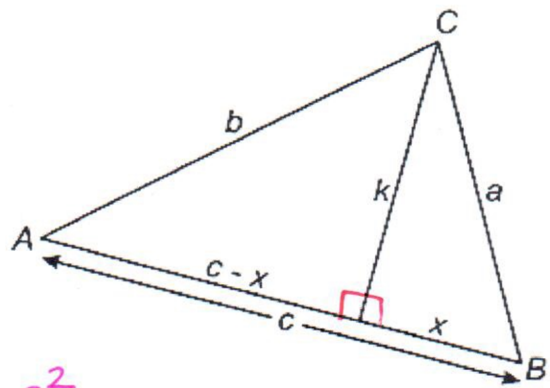
9-5 Notes: Law of Cosines and Area of a Triangle

Notes

LAW OF COSINES

Consider the triangle ΔABC. From vertex C, altitude k is drawn and separates side c into segments x and $c - x$. Why can the segments be written this way?

$c - x + x = c$



1. The altitude separates ΔABC into two right triangles. Use the Pythagorean Theorem to write two equations, one relating k, b, and $c - x$, and another relating a, k, and x.

$a^2 + b^2 = c^2$

$k^2 + (c-x)^2 = b^2$ $\quad$ $x^2 + k^2 = a^2$

2. Notice that both equations contain k^2. Solve each equation for k^2.

$k^2 = b^2 - (c-x)^2$ $\quad$ $k^2 = a^2 - x^2$

3. Since both of the equations in Question 2 are equal to k^2, they can be set equal to each other. Set them equal to each other to form a new equation.

$b^2 - (c-x)^2 = a^2 - x^2$

4. Notice that the equation in Question 3 involves x. However, x is not a side of ΔABC. As a result, we will attempt to rewrite the equation in Question 3 so that it does not include x. Begin by expanding $(c - x)^2$.

$(c-x)(c-x)$

$c^2 - cx - cx + x^2$

$c^2 - 2cx + x^2$

$b^2 - (c^2 - 2cx + x^2) = a^2 - x^2$

$b^2 - c^2 + 2cx \cancel{- x^2} = a^2 \cancel{- x^2}$

$\quad \cancel{+x^2} \quad \cancel{+x^2}$

$b^2 - c^2 + 2cx = a^2$

5. Solve the equation in Question 4 for b^2.

$b^2 \cancel{- c^2} + \cancel{2cx} = a^2$

$\quad \cancel{+c^2} \quad \cancel{-2cx} \quad +c^2$

$\quad -2cx$

$b^2 = a^2 + c^2 - 2cx$

6. The equation in Question 5 still involves x. To eliminate x from the equation, we will attempt to substitute an equivalent expression for x. Write an equation involving both $cosB$ and x. Why use $cosB$?

use adjacent side x.

$\cos B = \frac{x}{a}$

7. Solve the equation from Question 6 for x. Why solve for x? replace x.

$x = a\cos B$

8. Substitute the equivalent expression for x into the equation from Question 5. They resulting equation contains only sides and angles of ΔABC. This equation is called the *Law of Cosines*.

$b^2 = a^2 + c^2 - 2ac\cos B$

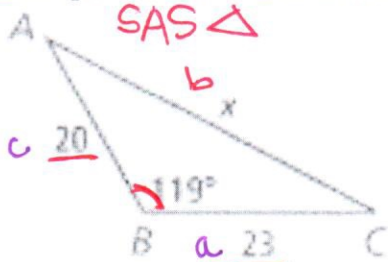
9. Using a similar method, two other forms of this law could be developed for a^2 and c^2. Based on your work from Questions 1-8, write two other forms of the Law of Cosines for ΔABC.

$a^2 = b^2 + c^2 - 2bc\cos A$

$c^2 = a^2 + b^2 - 2ab\cos C$

Examples: Use the Law of Cosines. Find the length of *x* to the nearest tenth.

SAS△



$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = (23)^2 + (20)^2 - 2(23)(20)(\cos 119°)$

$b^2 = 929 \cancel{-}+(-446.0249)$

$b^2 = 1375.0249$

$\boxed{b = 37.1}$

The sailboat race committee wants to lay out a triangular course with 40° angle between two sides that measure 3.5 miles and 2.5 miles. What will be the approximate length of the third side?

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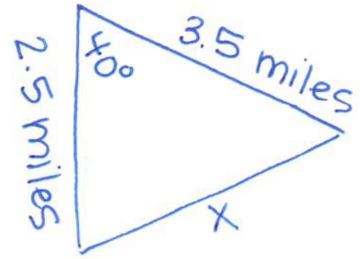


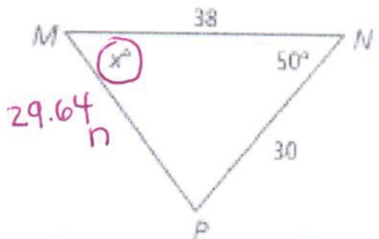
$x^2 = (3.5)^2 + (2.5)^2 - 2(3.5)(2.5)\cos 40°$

$x^2 = 18.5 - 13.4058$

$\sqrt{x^2} = \sqrt{5.0942}$

$\boxed{x = 2.3 \text{ miles}}$

Use both the Law of Cosines and the Law of Sines to solve for the missing information. Round your answer to the nearest tenth.



$n^2 = 38^2 + 30^2 - 2(38)(30)\cos 50°$

$n^2 = 2344 - 1465.556$

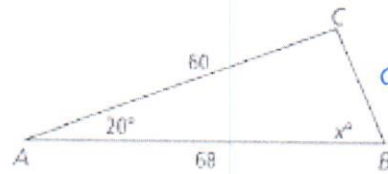
$n^2 = 878.444$

$n = 29.64$

$\frac{\sin x°}{30} = \frac{\sin 50°}{29.64}$ $\quad \frac{29.64 \sin x°}{29.64} = \frac{30 \sin 50°}{29.64}$

$\sin^{-1}(\sin x°) = \sin^{-1}(0.7753)$

$\boxed{x° = 50.8°}$



$a = 23.582$

$a^2 = 60^2 + 68^2 - 2(60)(68)\cos 20°$

$a^2 = 8224 - 7667.8918$

$a^2 = 556.1082$

$a = 23.582$

So

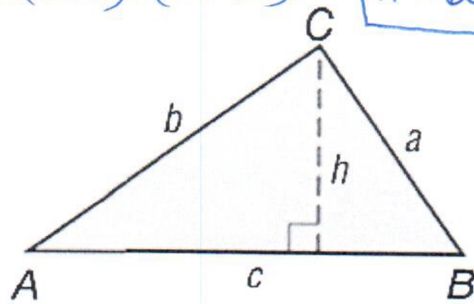
$\frac{\sin 20°}{23.582} = \frac{\sin x°}{60}$

$\frac{60 \sin 20°}{23.582} = \frac{23.582 \sin x°}{23.582}$

$\sin^{-1}(\sin x°) = \sin^{-1}(0.8702)$ $\quad \boxed{x° = 60.5°}$

AREA OF A TRIANGLE

In the triangle to the right, h represents the length of the altitude to side c in ΔABC. You can use this expression for the height of the triangle to develop a formula of the area of a triangle.



1. Start with the area formula for any triangle. Then replace values with what you know from the provided triangle.

   $A_\Delta = \frac{1}{2}bh$

   $A_\Delta = \frac{1}{2}ch$

2. Replace h with a trigonometric formula like in Law of Cosines.

   $\sin B = \frac{h}{a}$ $\quad h = a\sin B$

3. Simplify.

   $A_\Delta = \frac{1}{2}ac\sin B$

4. Using the equation in Question 3, we can develop the equivalent expressions for Area of a Triangle involving $\angle A$ and $\angle B$. **(These formulas only work when you have a triangle given SAS)**

   $A_\Delta = \frac{1}{2}ab\sin C$

   $A_\Delta = \frac{1}{2}bc\sin A$