Secondary 3 Honors – Unit 12 Homework Packet **Name: _____**

Period: _____

Practice 12-1: Parametric Equations

1. Kirby hits a ball when it is 4 feet above the ground with an initial velocity of 120 ft./sec. The ball leaves the bat at a 30° angle with the horizontal and heads toward a 30-ft fence, 350 feet away from home plate. Use the following parametric equations to answer the questions.

$$x = (120\cos 30°)t$$
$$y = -16t^2 + (120\sin 30°)t + 4$$

   a. Does the ball clear the fence?
   b. If so, by how much does it clear the fence? If not, could the ball be caught by the outfielder?

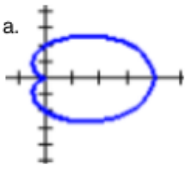
2. A golf ball is hit straight up from a height of 5 feet, with an initial velocity of 80 ft./sec. The following parametric equations are used to model this motion.

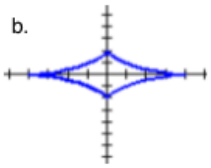
$$x = t$$
$$y = -16t^2 + 80t + 5$$

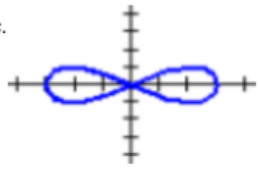
   a. Draw a graph of this motion.
   b. How high is the ball after 4 seconds?
   c. What is the maximum height of the ball?
   d. How many seconds does it take to reach its maximum height?

3. Match the parametric equation with its graph. Explain how you might approach the problem if you couldn't use a calculator.

$x = 4\cos^3 t$ $y = 2\sin^3 t$ | $x = 3\cos t$ $y = \sin 2t$ | $x = 2\cos t + 2\cos^2 t$ $y = 2\sin t + \sin 2t$

a. 

b. 

c. 

Graph each parametric equation by hand. Start by creating a T-chart.

4. $x = t + 2$
   $y = 1 + \frac{3}{t}$

5. $x = \cos t$
   $y = \sin t$

Eliminate the parameter and then identify the graph of the parametric curve.

6. $x = 1 + t$
   $y = t$

7. $x = t$
   $y = t^2 - 3$

8. $x = t^2$
   $y = t + 1$

9. $x = t$
   $y = t^3 - 2t + 3$

10. A rock is thrown straight up from level ground with its position above ground at any time, $t \geq 0$ given by the parametric equations:
$x = 5$
$y = -16t^2 + 80t + 7$
At what time, *t*, will the rock be 91 feet above the ground?

11. Draw the graph of the following parametric equations and then choose the best description of the graph.

$x = 1 - t$
$y = 3t + 2$
$t \geq 0$

a straight line
a line segment
a ray
a parabola
a circle

Practice 12-2: Polar Coordinates and Polar Graphs
Plot the points with the given polar coordinates.

1. $(3, 210°)$ **2.** $\left(2, -{}^{5\pi}/_{6}\right)$ **3.** $(-2, -30°)$

4. $\left(3, {}^{4\pi}/_{3}\right)$ **5.** $(-2, 120°)$ **6.** $(-3, 135°)$

Use algebra to find the rectangular coordinates of the point with given polar coordinates.

7. $\left(1.5, {}^{7\pi}/_{3}\right)$ **8.** $\left(-2, -{}^{14\pi}/_{5}\right)$

9. $\left(1, {}^{\pi}/_{2}\right)$ **10.** $(2, 270°)$

Convert the polar equations to rectangular form and identify the graph.

11. $r = 3\sec\theta$ **12.** $r\csc\theta = 1$

Convert the rectangular equation to polar form.

13. $(x-3)^2 + y^2 = 9$ **14.** $3x + 4y = 2$

Solve the following problems. Start by graphing the polar coordinates.

15. The location, given in polar coordinates, of two planes, flying at the same altitude, approaching the Salt Lake City airport are (4 mi, 12°) and (2 mi, 72°). Find the distance between the airplanes.

16. The location of two dementors, flying at the same altitude, from Azkaban Prison, given in polar coordinates, are (3 mi, 170°) and (5 mi, 150°). Find the distance between the dementors.

Practice 12-3: Polar Curves
Sketch the graph of the polar curve.

1. $r = 3$

2. $r = 2\sin 3\theta$

3. $r = -3\cos 4\theta$

4. $r = 6 - 5\cos\theta$

5. $r = 4 + 4\cos\theta$

6. $r = 3 - \sin\theta$

7. $r = 2 + 5\cos\theta$

8. $r^2 = \sin 2\theta\,, 0 \le \theta \le 2\pi$

9. $r^2 = 9\cos 2\theta\,, 0 \le \theta \le 2\pi$

10. Select the two equations whose graphs are the same curve. Then, even though the graphs of the equations are identical, describe how the two paths are different as θ increases from 0 to 2π.

$$r_1 = 1 + 3\sin\theta, \qquad r_2 = -1 + 3\sin\theta, \qquad r_3 = 1 - 3\sin\theta$$

11. True or False: The graph of $r = 2 + \cos\theta$ is symmetric about the x-axis. Justify your answer.

12. What is the number of petals on the rose curve $r = 3\cos 2\theta$? Justify your answer.
13. Describe the symmetry of the rose curve of $r = 3\cos 2\theta$. Justify your answer.
14. What is the number of petals on the rose curve $r = 5\sin 3\theta$? Justify your answer.

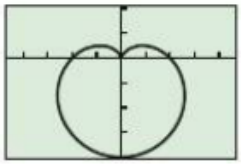
15. Match each of the equations with the correct polar curves, WITHOUT YOUR CALCULATOR. Justify your answer.

$r = 2 - 2\cos\theta$

$r = 2 + 3\cos\theta$

$r = 2 - 2\sin\theta$

$r = 2 - 1.5\sin\theta$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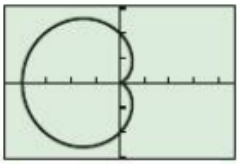
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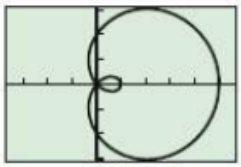
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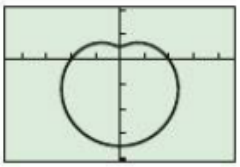
[–4.7, 4.7] by [–4.1, 2.1]
(d)

Practice 12-4: Polar Curves—Graphing by hand
Graph the following polar curves by hand. Show all work.

1. $r = 1 + 4\cos\theta$

2. $r^2 = 36\sin 2\theta$

3. $r = 2\sin\theta$

4. $r = 5\cos 3\theta$

5. $r = 3 - 3\sin\theta$

6. $r = 5\sin 3\theta$

7. A graph known as Archimedes spiral is created by graphing $r = a + b\theta$, where *a* and *b* are numbers, either positive or negative. By adjusting *a* and *b*, what is the greatest number of curves you can created in a $[-30, 30]$ by $[-20, 20]$ window?

Practice 12-5: Complex Coordinates and Complex Numbers

Graph each number in the complex plane and find its absolute value.

1. $z = 4 + 4i$ **2.** $z = -4 - 6i$ **3.** $z = 3 + 4i$

4. $z = -3 - 7i$ **5.** $z = -7 + 5i$ **6.** $z = 2 - 5i$

Express each complex number in polar form.

7. $4 + 4i$ **8.** $-2 + i$ **9.** $4 - \sqrt{2}i$

10. $2 - 2i$ **11.** $4 + 5i$ **12.** $-1 - \sqrt{3}i$

Graph the complex polar equation. Then convert the equation to rectangular form.

13. $10(\cos 6 + i \sin 6)$ **14.** $2\left(\cos\frac{4\pi}{3} + i \sin\frac{4\pi}{3}\right)$

Find each product or quotient. Then express the answer in rectangular form.

15. $6\left(\cos\frac{\pi}{2} + i \sin\frac{\pi}{2}\right) \cdot 4\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$ **16.** $2(\cos 90° + i \sin 90°) \cdot 2(\cos 270° + i \sin 270°)$

17. $4\left(\cos\frac{9\pi}{4} + i \sin\frac{9\pi}{4}\right) \div 2\left(\cos\frac{3\pi}{2} + i \sin\frac{3\pi}{2}\right)$ **18.** $6\left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right) \div 2\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$

Review!

Find the polar coordinates of the points, given the rectangular coordinates.

19. $\left(-\sqrt{3}, 3\right)$ **20.** $(-2, 5)$

Practice 12-6: DeMoivre's Theorem

Find each power, and then express you answer in rectangular form.

1. $\left(2 + 2\sqrt{3}i\right)^6$ **2.** $(-5 + 12i)^3$ **3.** $\left(\sqrt{3} - i\right)^3$

4. $(2 + 4i)^4$ **5.** $(2 + 3i)^2$ **6.** $\left[2\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)\right]^4$

Find all the distinct pth roots of the complex number.

7. sixth roots of i **8.** fifth roots of $-i$

9. cube roots of $-117 + 44i$ **10.** square root of $-3 - 4i$

Review!

11. Find the quotient. Then express your answer in rectangular form.

$$5\left(\cos\frac{\pi}{2} + i \sin\frac{\pi}{2}\right) \div \frac{1}{3}\left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right)$$