Name: ______________________ Period: A3

1.2 Notes Arithmetic Series

Warm-Up: For each of the following sequences, determine whether it is arithmetic, geometric, or neither. If it is arithmetic, find the common difference. If it is geometric, find the common ratio.

1. 2, 4, 8, 16, . . .   ×2 ×2 ×2   geometric $r=2$
2. $2^3, 2^7, 2^{11}, 2^{15}, \ldots$ $\times 2^4$ $\times 2^4$ $\times 2^4$ geometric $r=2^4$
3. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$
4. 1, 5, 9, 13, 17, . . .

ARITHMETIC: starting value of a and a common difference of d.

Recursive:

$a_1 = a$

$a_n = a_{n-1} + d$

Explicit:

$a_n = a_1 + (n-1)d$

5. What is the 100th term of the arithmetic sequence that begins 6, 11, . . . ? $n=100$ a_1 $+5$

$a_{100} = 6 + (100-1)(5) = 6 + (99)(5) = \boxed{501}$

6. What are the second and third terms of the arithmetic sequence 100, 94, 88, 82, . . . ? a_1 a_4

$a_n = a_1 + (n-1)(d)$

$82 = 100 + (4-1)(d)$ $d = -6$

$82 = 100 + 3d$

GEOMETRIC: a starting value of a and a common ratio of r.

Recursive:

$a_1 = a$

$a_n = a_{n-1} \cdot r$

Explicit:

$a_n = a_1 \cdot r^{n-1}$

7. What is the 10th term of the geometric sequence 4, 12, 36, . . . ? $n=10$ a_1 ×3 ×3 $r=3$

$a_n = a_1 \cdot r^{n-1}$

$a_{10} = 4(3)^{10-1} = 4(3)^9 = \boxed{78732}$

8. What are the second and third terms of the geometric sequence 2, -6, 18, -54, . . . ? a_1 a_4

$-54 = 2 \cdot r^{4-1}$

$\frac{-54}{2} = \frac{2r^3}{2}$ $\sqrt[3]{-27} = \sqrt[3]{r^3}$ $r = -3$

9. When radioactive substances decay, the amount remaining will form a geometric sequence when measured over constant intervals of time. The table shows the amount of Np-240, a radioactive isotope of Neptunium, initially and after 2 hours. What are the amounts left after 1 hour, 3 hours, and 4 hours?

Hours Elapsed	0	1	2	3	4
Grams of Np-240	1244	■	346	■	■

a_1 a_3

$346 = 1244 \cdot r^{3-1}$

$\sqrt{\frac{346}{1244} = \frac{\cancel{1244} r^2}{\cancel{1244}}}$

$r = 0.527$

$a_2 = 1244(0.527)^{2-1}$

$a_2 = \boxed{655.588 \text{ 1st hour}}$

OR $1244(0.527)$

$\boxed{\text{3rd hour } 182.076}$

$\boxed{\text{4th hour } 95.954}$

ARITHMETIC SERIES

Series: the SUM of all terms in a sequence

Finite Series: first AND last term

Infinite Series: continues without end

Find the sum of each finite arithmetic series:

$4+9+14+19+24+\ldots+89+94+99$

103

103

103

10 sums

$10 \times 103 = \boxed{1030}$

$7+14+21+\ldots+91+98+105$

The Sum of a Finite Arithmetic Series: $a_1 + a_2 + a_3 + \ldots + a_n$

$$\star\ S_n = \frac{n}{2}(a_1 + a_n)$$

n = # of terms a_n = last term

a_1 = first term S_n = sum

Is it necessary to know the common difference when using the formula to find the value of a finite arithmetic series? Explain. you need the common diff. to find the # of terms

Find the sum of this finite arithmetic series: $-3 + -6 + -9 + \ldots + -30$

$-30 = -3 + (n-1)(-3)$

$+3 \quad +3$

$\frac{-27}{-3} = \frac{(n-1)(-3)}{-3}$

$n-1 = 9$

$+1 \quad +1$

$n = 10$

$S_{10} = \frac{10}{2}(-3 + -30)$

$= 5(-33) = \boxed{-165}$

10. A company pays a $10,000 bonus to salespeople at the end of their first 50 weeks if they make 10 sales in their first week, and then improve their sales numbers by two each week thereafter. One salesperson qualified for the bonus with the minimum possible number of sales. How many sales did the salesperson make in week 50? In all 50 weeks?

a_1 $d=2$

a_{50} S_{50}

$a_1 = 10$

$a_2 = 12$

$a_3 = 14$

$a_{50} = 10 + (50-1)(2)$

$= 10 + (49)(2)$

$= \boxed{108}$

$S_{50} = \frac{50}{2}(10 + 108)$

$= 25(118)$

$= \boxed{2950 \text{ total sales}}$

Summation Notation:

$$\sum_{n=}^{n=} \text{an formula}$$

Example: $4 + 9 + 14 + \cdots + 34$

What do you need in order to write a series in summation notation?

explicit formula

upper limit (where the series ends)

lower limit (where the series starts)

What is summation notation for the series: $7 + 11 + 15 + \ldots + 203 + 207$?

$a_n = a_1 + (n-1)(d)$

$a_n = 7 + (n-1)(4)$

$= 7 + 4n - 4 = 4n + 3$

$$\boxed{\sum_{n=1}^{n=51} 4n+3}$$

$4n+3 = 7$

$-3 \quad -3$

$4n = 4$

$n = 1$ lower limit

$4n+3 = 207$

$-3 \quad -3$

$4n = 204$

$n = 51$ upper limit

Can the series be written correctly in summation notation in more than one way? Explain.

yes. you can start your limits at different places

$\sum_{n=0} \quad \sum_{n=1}$

Finding the Sum of a Series

What is the sum of the series written in summation notation? $S_n = \frac{n}{2}(a_1 + a_n)$

$$\sum_{n=1}^{70}(5n+3)$$

$a_1 = 5(1)+3 = 8$
$a_{70} = 5(70)+3 = 353$
$n = 70$

$S_{70} = \frac{70}{2}(8+353)$

$\boxed{12635}$

Your turn!

Find the sum of the series written in summation notation.

even $$\sum_{n=1}^{40}(3n-8)$$

$a_1 = 3(1)-8 = -5$
$a_{40} = 3(40)-8 = 112$
$n = 40$

$S_{40} = \frac{40}{2}(-5+112) = \boxed{2140}$

$$\sum_{n=1}^{4} n^3 = 1^3 + 2^3 + 3^3 + 4^3$$
$1 + 8 + 27 + 64 = \boxed{100}$

odd $$\sum_{n=1}^{7}(5n+6)$$

$a_1 = 5(1)+6 = 11$
$a_7 = 5(7)+6 = 41$

$S_7 = \frac{7}{2}(11+41) = \boxed{182}$

$$\sum_{n=0}^{100}(-1)^n = (-1)^0 + (-1)^1 + (-1)^2 + (-1)^3 + \dots + (-1)^{98} + (-1)^{99} + (-1)^{100}$$
$\boxed{1 + -1} + \boxed{1 + -1} + \dots + \boxed{1 + -1} + 1 = \boxed{1}$

$$\sum_{n=2}^{11}(-2n+3)$$

$a_2 = -2(2)+3 = -1$
$a_{11} = -2(11)+3 = -19$
$n = 10$

$S_{10} = \frac{10}{2}(-1+-19) = \boxed{-100}$

Stamp Question!!

Turn into basket on a separate sheet of paper.

a. A supermarket displays cans in a triangle, like the one shown, with two cans in the top row. Write an explicit formula for the sequence of the number of cans.
b. Use summation notation to write the related series for a triangle with 10 cans in the bottom row.
c. Suppose the triangle had 17 rows. How many cans would be in the 17th row?
d. Could the triangle have 110 cans total? 140 cans? Explain.