Name: ______________________________ Period: _____

## 1.3 Notes Geometric Series

### GEOMETRIC SERIES

### FINITE SERIES

$a_1$: first term

$r$: common ratio

$n$: # of terms

$a_1 \cdot r \cdot r$

Sum of Finite Geometric Series: $a_1 + a_1 r + a_1 r^2 + \ldots + a_1 r^{n-1}, r \neq 1$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

What is the sum of the finite geometric series: $3 + 6 + 12 + 24 + \ldots + 3072$ where n=11?

$n=11$, $r=2$, $a_1=3$

$$S_{11} = \frac{3(1-2^{11})}{1-2} = \boxed{6141}$$

Find the sum of the finite geometric series:

$$\sum_{n=0}^{20} 4\left(\frac{1}{2}\right)^n$$

$\uparrow r$

$n=21$

$a_1 = 4\left(\frac{1}{2}\right)^0 = 4$

$r = 1/2$

$$S_{21} = \frac{4(1-1/2^{21})}{1-1/2} = \boxed{8.0}$$

$$\sum_{n=1}^{12} \frac{1}{3}(-4)^n$$

$\uparrow r$

$n=12$

$a_1 = \frac{1}{3}(-4)^1 = -4/3$

$r = -4$

$$S_{12} = \frac{-\frac{4}{3}(1-(-4)^{12})}{1-(-4)} = \boxed{4473924}$$

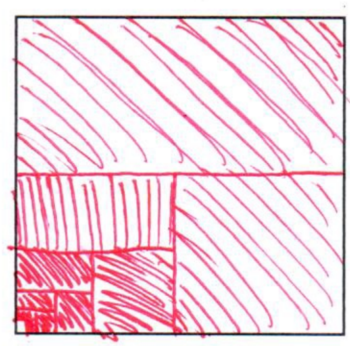
### INFINITE SERIES

1. Use the square to the right to model the geometric series:

$$\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots + \left(\frac{1}{2}\right)^n + \ldots$$

$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

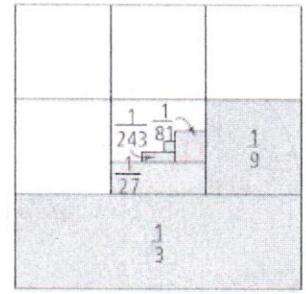
$r = \frac{1}{2}$



Shade one half of the square. Then shade one half of the remaining unshaded region. Then shade one half of that remaining unshaded region. Continue this pattern.

According to your geometric figure, what does the sum of the series appear to be? 1

2. Write the series modeled by the figure at the right. Evaluate the sum of the series.




$$\frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \ldots + \left(\frac{1}{3}\right)^n + \ldots$$

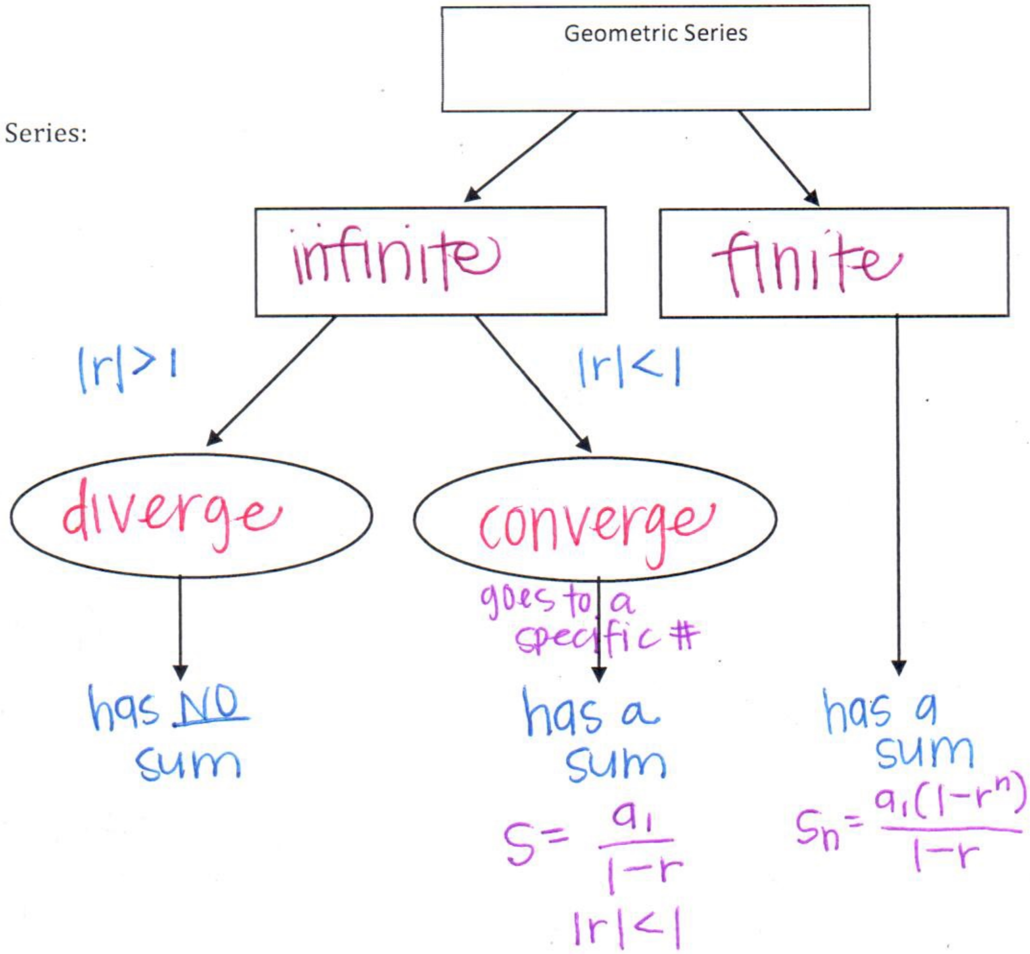
$r = \frac{1}{3}$ sum $= \frac{1}{2}$

What do all of these series have in common?

Can an infinite series have a sum?

yes, IF $|r| < 1$

Analyzing Infinite Geometric Series:




Does the series converge or diverge? If it converges, what is the sum?

| | |
|---|---|
| 1. $\frac{1}{2}+\frac{3}{4}+\frac{9}{8}+\ldots$ <br> x3 x3 <br> x2 x2 <br> diverges <br> $r=\frac{3}{2}>1$ | 2. $\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}+\ldots$ <br> x−1 x−1 x−1 <br> x3 x3 x3 <br> converges <br> $r=-\frac{1}{3}$ <br> $\lvert -1/3 \rvert=\frac{1}{3}<1$ <br> $S=\frac{1/3}{(1-(-1/3))}=\frac{1/3}{4/3}=\boxed{1/4}$ |
| 3. $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ <br> ↑r <br> converges <br> $r=\frac{2}{3}<1$ <br> $a_1=\left(\frac{2}{3}\right)^1$ <br> $S=\frac{2/3}{1-2/3}=\boxed{2}$ | 4. $\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)\left(-\frac{5}{4}\right)^n$ <br> ↑r <br> diverges <br> $r=-\frac{5}{4}$ <br> $\lvert -\frac{5}{4} \rvert=\frac{5}{4}>1$ |