Name: ______________________________ Period: _____

1.4 Binomial Theorem

For each of the following tasks, work with your group to find the answers. We are going to see what kind of connections we can find between different parts of mathematics! Yay!!

TASK #1

Your job is to multiply and find all the terms in $(x+1)^4$.

Recall that this means $(x+1)(x+1)(x+1)(x+1)$.

Start by multiplying: $(x+1)(x+1)$. Write your answer in the space below.

$x^2+x+x+1 = x^2+2x+1$

Now, multiply this answer by $(x+1)$, combine like terms, write your answer in the space below. $(x^2+2x+1)(x+1) = x^3+x^2+2x^2+2x+x+1$

x^3+3x^2+3x+1

Finally, multiply this result by $(x+1)$, combine like terms, write your answer in the space below. $(x^3+3x^2+3x+1)(x+1)$

$x^4+x^3+3x^3+3x^2+3x^2+3x+x+1 = x^4+4x^3+6x^2+4x+1$

Make sure your answer in written in the correct order. Highest powers of x should come first, down to the lowest powers.

What are the coefficients of x in your answer? Write these coefficients in the boxes below.

1	4	6	4	1

Now for the tricky part: Multiply and find all of the terms of $(x+y)^4$. Don't forget that to combine-like terms, the exponents on the variables must be the same!

$x^4+4x^3y+6x^2y^2+4xy^3+y^4$

TASK #2

In how many ways can 4 coins be tossed? For example, one way is Tails, Heads, Heads, Tails. Using T's to represent tails, and H's to represent heads, write all of the possibilities for this situation below.

TTTT	TTHH	THHH	HHHH
TTTH	THTH	HTHH	
TTHT	THHT	HHTH	
THTT	HTHT	HHHT	
HTTT	HHTT	HTTH	

How many possibilities did you find?

16

If 4 coins are tossed, how many ways are there to get 0 heads?

1

If 4 coins are tossed, how many ways are there to get 1 heads?

4

If 4 coins are tossed, how many ways are there to get 2 heads?

6

If 4 coins are tossed, how many ways are there to get 3 heads?

4

If 4 coins are tossed, how many ways are there to get 4 heads?

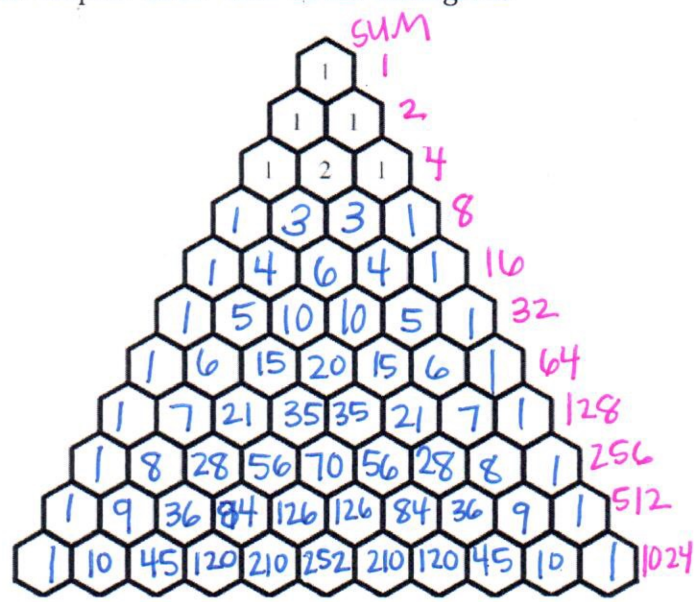
1

Write the answers to these questions, in order, in the boxes below.

1	4	6	4	1

TASK #3

Below is a blank version of a famous mathematical pattern, Pascal's Triangle. It has been studied for hundred of years by mathematicians and students and contains many interesting patterns. The concept behind the triangle is simple: each "cell" in the triangle is the sum of the two numbers above it. The first few rows have been filled in for you. Your job is to fill in the remaining rows of the triangle.



Question 1. What is the sum of the entries for each row of the triangle? Find each sum and write it at the end of the row. How are these numbers related?

$r=2$

Questions 2. What are the entries in row 4 of the triangle? Row 4 is the row which has 5 entries (note: the top row is considered row 0). Write your answer in the boxes below.

1	4	6	4	1

Question 3. Describe patterns that exist along the different diagonals in the triangle.

Question 4. Write a formula for the sum of the terms in any particular row.

2^n

CONNECTIONS

Now that you have explored each task and discussed the results, take a step back and note the connections that exist between the tasks. With your group, write a sentence or two which describes the connections between each of the following:

- Coin flipping and Pascal's triangle

- Multiplying binomials and Pascal's triangle

- Coin Flipping and Multiplying Binomials

Pascal's triangle can be used in many different ways in mathematics. Its patterns describe combinations, binomial expansions, arrangement of objects, and more! For today though, we are only going to focus on **binomial expansion**.

A binomial is an algebraic expression of the sum or difference of two or more terms. When we talk about expanding a binomial that means raising it to a power and writing out each individual term. Your work on Task #1 was a binomial expansion.

We can use the pattern of coefficients and the pattern $a^n, a^{n-1}b, a^{n-2}b^2, \ldots, a^2b^{n-2}, ab^{n-1}, b^n$ to write the expansion of $(a+b)^n$.
Consider the expansions of $(a+b)^n$ for the first few values of n.

Row	Power	Expanded Form	Coefficients Only
0	$(a+b)^0$	1	1
1	$(a+b)^1$	$1a^1 + 1b^1$	1 1
2	$(a+b)^2$	$1a^2 + 2a^1b^1 + 1b^2$	1 2 1
3	$(a+b)^3$	$1a^3 + 3a^2b^1 + 3a^1b^2 + 1b^3$	1 3 3 1
4	$(a+b)^4$	$1a^4 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1b^4$	1 4 6 4 1

The "coefficients only" column matches the numbers in Pascal's Triangle.
Using Pascal's Triangle, we can more easily write the binomial expansion of any binomial.

Example #1:

What is the expansion of $(a+b)^6$? Use Pascal's Triangle. 1 6 15 20 15 6 1

The exponents for a begin with 6 and decrease to 0.

$$1a^6b^0 + 6a^5b^1 + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6a^1b^5 + 1a^0b^6$$

The exponents for b begin with 0 and increase to 6.

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6.$$

Now you try!

1 8 28

1. What is the expansion of $(a+b)^8$? Use Pascal's Triangle.

$1(a)^8(b)^0+8(a)^7(b)^1+28(a)^6(b)^2+56(a)^5(b)^3+70(a)^4(b)^4+56(a)^3(b)^5+28(a)^2(b)^6+8(a)^1(b)^7$

$+1(a)^0(b)^8 = \boxed{a^8+8a^7b+28a^6b^2+56a^5b^3+70a^4b^4+56a^3b^5+28a^2b^6+8ab^7+b^8}$

2. Expand the binomial: $(x+2)^5$ 1 5 10 10 5 1

$1x^5 2^0+5x^4 2^1+10x^3 2^2+10x^2 2^3+5x^1 2^4+1x^0 2^5$

$\boxed{x^5+10x^4+40x^3+80x^2+80x+32}$

3. Expand the binomial: $(x-5)^3$. 1 3 3 1

$1x^3(-5)^0+3x^2(-5)^1+3x^1(-5)^2+1x^0(-5)^3$

$\boxed{x^3-15x^2+75x-125}$

The Binomial Theorem gives the general formula for expanding a binomial.

BINOMIAL THEOREM: For every positive integer n,

$$(a+b)^n = P_0a^n + P_1a^{n-1}b + P_2a^{n-2}b^2 + \ldots + P_{n-1}ab^{n-1} + P_nb^n$$

Where $P_0, P_1, \ldots, P_n$ are the numbers in the nth row of Pascal's Triangle.

Example #2:

What is the expansion of $(3x-2)^5$? Use the Binomial Theorem.

Think	Write
For $(3x-2)^5$, use the 5th row of Pascal's Triangle.	Pascal's Triangle 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1
The Binomial Theorem uses a binomial sum.	$(3x-2)^5 = (3x+(-2))^5$
Apply the Binomial Theorem.	$= (3x)^5 + 5(3x)^4(-2)^1 + 10(3x)^3(-2)^2 + 10(3x)^2(-2)^3 + 5(3x)^1(-2)^4 + 1(-2)^5$
Simplify.	$= 243x^5 - 810x^4 + 1080x^3 - 720x^2 + 240x - 32$

Your turn!

1 4 6 4 1

1. What is the expansion of $(2x-3)^4$? Use the Binomial Theorem.

$1(2x)^4(-3)^0+4(2x)^3(-3)^1+6(2x)^2(-3)^2+4(2x)^1(-3)^3+1(2x)^0(-3)^4$

$2^4x^4 \quad 2^3x^3 \quad 2^2x^2 \quad 2^1x^1 \quad 2^0x^0$

$\boxed{16x^4-96x^3+216x^2-216x+81}$

2. Expand $(3a-7)^3$ 1 3 3 1

$1(3a)^3(-7)^0+3(3a)^2(-7)^1+3(3a)^1(-7)^2+1(3a)^0(-7)^3$

$\boxed{27a^3-189a^2+441a-343}$