- Learning division:
  - ↳ to prepare to use the Rational Roots Thm
  - ↳ to find upper and lower bounds for zeros of polynomials (x-intercepts)
- Gives us new and better ways to factor polynomials

Name: ______________________________ Period: ______

Secondary 3 Honors
2-3 Notes Real Zeros with Long and Synthetic Division

Divide the two functions using LONG DIVISION and write a summary statement in fraction form.

1. $(5x^5 - 4x^4 - 35x + 35) \div (5x - 4)$

$x^4 - 7$

$5x-4 \,\big)\, 5x^5 - 4x^4 + 0x^3 + 0x^2 - 35x + 35$

$-(5x^5 - 4x^4)$

$\cancel{0x^4 + 0x^3 + 0x^2} - 35x + 35$

$-(-35x + 28)$

7

$\boxed{x^4 - 7 + \frac{7}{5x-4}}$

2. $(2r^5 + 8r + 16r^2 - 11 - 9r^4 - 12r^3) \div (2r - 1)$

$r^4 - 4r^3 - 8r^2 + 4r + 6$

$2r-1 \,\big)\, 2r^5 - 9r^4 - 12r^3 + 16r^2 + 8r - 11$

$-(2r^5 - r^4)$

$-8r^4 - 12r^3$

$-(-8r^4 + 4r^3)$

$-16r^3 + 16r^2$

$-(-16r^3 + 8r^2)$

$8r^2 + 8r$

$-(8r^2 - 4r)$

$12r - 11$

$-(12r - 6)$

-5

$\boxed{r^4 - 4r^3 - 8r^2 + 4r + 6 + \frac{-5}{2r-1}}$

Divide using SYNTHETIC DIVISION, and write a summary statement in fraction form.

3. $(n^3 - 3n^2 - 73n - 25) \div (n + 7)$

zero of divisor

divisor

-7	1	-3	-73	-25
+	↓	-7	70	21
	1	-10	-3	-4
	n^2	n	c	R

$\boxed{n^2 - 10n - 3 + \frac{-4}{n+7}}$

4. $(x^5 - 3x^4 - x) \div (x - 3)$

3	1	-3	0	0	-1	0
+	↓	3	0	0	0	-3
	1	0	0	0	-1	-3
	x^4	x^3	x^2	x	c	R

$\boxed{x^4 - 1 + \frac{-3}{x-3}}$

5. $(n^5 - 5n^4 - 5n^2 + 27n) \div (n - 5)$

5	1	-5	0	-5	27	0
+	↓	5	0	0	-25	10
	1	0	0	-5	2	10
	n^4	n^3	n^2	n	c	R

$\boxed{n^4 - 5n + 2 + \frac{10}{n-5}}$

6. $(r^5 + 17r^4 + 78r^3 + 45r^2 - 88r - 55) \div (r + 9)$

-9	1	17	78	45	-88	-55
+	↓	-9	-72	-54	81	63
	1	8	6	-9	-7	8
	r^4	r^3	r^2	r	c	R

$\boxed{r^4 + 8r^3 + 6r^2 - 9r - 7 + \frac{8}{r+9}}$

Real Zeros of a Polynomial

Remainder Theorem: If a polynomial $f(x)$ is divided by $(x-k)$, then the remainder is

$\boxed{r = f(k).}$

k= zero of divisor

$x-k=0$
$+k \quad +k$ $\quad x=k$

Examples: Use the remainder theorem to find the remainder when $f(x)$ is divided by $(x-k)$.

- $f(x) = 3x^2 + 7x - 20; k = 2$ $(x-2)$

$r = f(2) = 3(2)^2 + 7(2) - 20$

$12 + 14 - 20$

$\boxed{r = 6}$

- $f(x) = 5x^2 + 8x + 12; k = -1$

$r = f(-1) = 5(-1)^2 + 8(-1) + 12$

$5 - 8 + 12$

$\boxed{r = 9}$

- $f(x) = 4x^3 + 6x^2 + x + 9; k = 3$

$r = f(k) = 4(3)^3 + 6(3)^2 + (3) + 9$

$108 + 54 + 3 + 9$

$\boxed{r = 174}$

Rational Zeros (Roots) Theorem: Tells us how to make a list of all potential rational zeros for a polynomial with integer coefficients.

$\dfrac{\text{factors of constant}}{\text{factors of leading coef.}}$

Examples: Use the Rational Zeros Theorem to write a list of all potential rational zeros. Then determine which ones, if any, are zeros. $r = 0$

- $f(x) = 3x^3 + 4x^2 - 5x - 2$

$\dfrac{\pm 1, \pm 2}{\pm 1, \pm 3} = \boxed{\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}}$

$$\begin{array}{r|rrrr} 1 & 3 & 4 & -5 & -2 \\ + & \downarrow & 3 & 7 & 2 \\ \hline & 3 & 7 & 2 & \boxed{0} \end{array}$$

$(3x^2 + 7x + 2)(x-1)$

$x^2 + 7x + 6$

$(x+1)(x+6)$

$(3x+1)(3x+6)$

$\boxed{(3x+1)(x+2)(x-1)}$

☆ $\boxed{\text{zeros}: -1/3, -2, 1}$ ☆

- $f(x) = x^3 - 3x^2 + 1$

$\dfrac{\pm 1}{\pm 1} = \boxed{\pm 1}$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 0 & 1 \\ + & \downarrow & 1 & -2 & -2 \\ \hline & 1 & -2 & -2 & \boxed{-1} \end{array}$$ X

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 0 & 1 \\ + & \downarrow & -1 & 4 & -4 \\ \hline & 1 & -4 & 4 & \boxed{-3} \end{array}$$ X

$\boxed{\text{NO RATIONAL ZEROS}}$

- $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$

$$\frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2} = \boxed{\pm 1, \pm \tfrac{1}{2}, \pm 2, \pm 4, \pm 8}$$

$$\begin{array}{r|rrrrr} -2 & 2 & -7 & -8 & 14 & 8 \\ + & \downarrow & -4 & 22 & -28 & 28 \\ \hline & 2 & -11 & 14 & -14 & \boxed{36} \end{array} \quad X$$

$$\begin{array}{r|rrrrr} 8 & 2 & -7 & -8 & 14 & 8 \\ + & \downarrow & 16 & 72 & & \\ \hline & 2 & 9 & 64 & & \end{array} \quad X$$

$$\begin{array}{r|rrrrr} 1 & 2 & -7 & -8 & 14 & 8 \\ + & \downarrow & 2 & -5 & -13 & 1 \\ \hline & 2 & -5 & -13 & 1 & \boxed{9} \end{array} \quad X$$

$$\begin{array}{r|rrrrr} -1/2 & 2 & -7 & -8 & 14 & 8 \\ + & \downarrow & -1 & 4 & 2 & -8 \\ \hline & 2 & -8 & -4 & 16 & \boxed{0} \end{array} \quad \checkmark$$

$$\underbrace{2x^3 - 8x^2}\underbrace{- 4x + 16}$$

~~choose another zero~~

$$2x^2(x-4) - 4(x-4)$$

$$(x-4)(2x^2-4)(x+1/2)$$

$$\boxed{x = 4, -1/2}$$

$$2x^2 - 4 = 0$$
$$+4 \quad +4$$
$$\frac{2x^2}{2} = \frac{4}{2}$$
$$\sqrt{x^2} = \sqrt{2}$$
$$x = \pm\sqrt{2}$$

irrational