Name: ______________________________

Secondary 3 Honors
2-6 Notes: Complex Zeros and Fundamental Theorem of Algebra

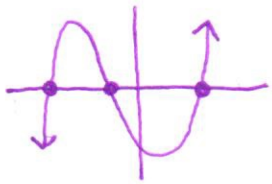
Fundamental Theorem of Algebra: Any polynomial of degree **n** ... has **n** roots (zeros)!

Examples: Let's see if that is true... Graph the following on your calculator and see how many zeros each has!

degree: 3

- $f(x) = x^3 + 6x^2 - 13x - 42$

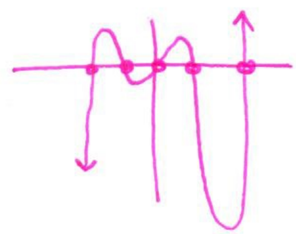
3 zeros



n=5

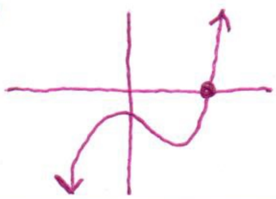
- $f(x) = x^5 - 7x^4 - 23x^3 + 39x^2 + 54x$

5 roots



- $f(x) = x^3 - 5x^2 + 2x - 10$

n=3



1 root/zero

2 roots are imaginary

COMPLEX ZEROS

Complex Zeros: If $a + bi$ is a zero of a polynomial function $f(x)$, then its complex conjugate $a - bi$ is also a zero of $f(x)$.

Examples: Find the polynomial from the given zeros

- $-3, 4, 2 - i$, $2+i$

$x=-3 \quad x=4 \quad x=2-i \quad x=2+i$

$[(x+3)(x-4)][(x-2-i)(x-2+i)]$

$[x^2-4x+3x-12][x^2-2x+\cancel{ix}-2x+4-\cancel{2i}-\cancel{ix}+\cancel{2i}-i^2]$

$-(-1)$

$[x^2-x-12][x^2-4x+4+1]$

$[x^2-x-12][x^2-4x+5]$

$x^4-4x^3+5x^2-x^3+4x^2-5x-12x^2+48x-60$

$\boxed{x^4-5x^3-3x^2+43x-60}$

- $0, 1, 1 + i$, $1-i$

$x=0 \quad x=1 \quad x=1+i \quad x=1-i$

$[(x)(x-1)][(x-1+i)(x-1-i)]$

$[x^2-x][x^2-x-\cancel{ix}-x+1+\cancel{i}+\cancel{ix}-\cancel{i}-i^2]$

$-(-1)$

$[x^2-x][x^2-2x+2]$

$x^4-2x^3+2x^2-x^3+2x^2-2x$

$\boxed{x^4-3x^3+4x^2-2x}$

- 2 (mult: 2), 3+i

$x=2 \quad x=2 \quad x=3+i \quad x=3-i$

$[(x-2)(x-2)][(x-3+i)(x-3-i)]$

$[x^2-2x-2x+4][x^2-3x\cancel{-ix}-3x+9\cancel{+3i}\cancel{+ix}\cancel{-3i}-i^2]$

$-(-1)$

$[x^2-4x+4][x^2-6x+10]$

$x^4-6x^3+10x^2-4x^3+24x^2-40x+4x^2-24x+40$

$\boxed{x^4-10x^3+38x^2-64x+40}$

Are zeros always the same as x-intercepts??? **NO** complex zeros are **NOT** x-intercepts

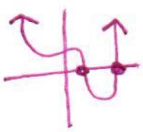
$4 \rightarrow 4+0i$

State the number of complex zeros and how many of those are real zeros.

- $f(x) = x^4 - 5x^3 + x^2 - 3x + 6$

$n=4$ FTofA

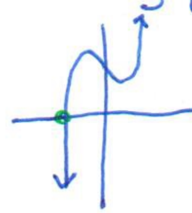
4 complex roots



2 real roots

- $f(x) = x^5 - 2x^2 - 3x + 6$

5 complex zeros



1 real zero

Examples: Find the zeros and write in factored form. Describe the end behavior using limits, find the degree and zeros (real and complex) of the function. Then graph the polynomial. BY HAND! :)

- $f(x) = x^3 - 6x^2 - 19x + 84$

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm7, \pm12, \pm14, \pm21, \pm28, \pm42, \pm84$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & -19 & 84 \\ + & \downarrow & 4 & -8 & -108 \\ \hline & 1 & -2 & -27 & \boxed{-24} \end{array} \quad X$$

$$\begin{array}{r|rrrr} 7 & 1 & -6 & -19 & 84 \\ + & \downarrow & 7 & 7 & -84 \\ \hline & 1 & 1 & -12 & \boxed{0} \end{array} \quad \checkmark$$

$(x-7)(x^2+x-12)$

$\boxed{f(x)=(x-7)(x+4)(x-3)}$

$x=7 \quad x=-4 \quad x=3$

odd, +

$$\boxed{\lim_{x\to-\infty} f(x) = -\infty \qquad \lim_{x\to\infty} f(x) = \infty}$$

degree: 3

$x = 7, -4, 3$

mult 1 cross, mult 1 cross, mult 1. cross