

3.1 Notes: Simplifying, Multiplying and Dividing Rational Expressions

Rational Expression: A fraction in which the numerator and/or the denominator are polynomials.

Examples of rational expressions:

$$\frac{2a}{4a^2} \quad \frac{3c}{6c^2}$$

$$\frac{2b^2}{3^2} \quad \frac{a+3}{2^2}$$

Excluded Values (also known as domain restrictions):

Value that make denominator zero.

Finding the excluded values: set denom. $\neq 0$.
Solve for x.

$$\frac{10x^2}{x^2(x+9)}$$

$x^2 \neq 0$
 $x \neq 0$

$x+9 \neq 0$
 $x \neq -9$

$$\frac{5}{x^2+7x+12}$$

$(x+3)(x+4)$

$x+3 \neq 0$
 $x \neq -3$

$x+4 \neq 0$
 $x \neq -4$

Simplifying Rational Expressions: Simplify each of the following and state the excluded values.

$$\frac{60x^6 \neq 0}{60} \cdot \frac{18x^4}{60x^6}$$

$x^6 \neq 0$
 $x \neq 0$

$$\frac{18xxxxx}{60xxxxxx} = \frac{3}{10x^2}$$

$$\frac{56x^6}{48x^5}$$

~~xxxxxx~~
~~xxxxxx~~

$$\frac{7x}{6}$$

$48x^5 \neq 0$
 $x \neq 0$

$$\frac{56a - 56}{16a}$$

$16a \neq 0$
 $a \neq 0$

$$\frac{7(a-1)}{2a}$$

$$\frac{2p^2 + 10p}{p + 5}$$

$p+5 \neq 0$
 $p \neq -5$

$$\frac{2p(p+5)}{p+5} = 2p$$

$$\frac{k-8}{3k^2 - 24k}$$

$3k \neq 0$
 $k \neq 0$

$k-8 \neq 0$
 $k \neq 8$

$$\frac{k-8}{3k(k-8)}$$

$$\frac{r+10}{r^2 + 15r + 50}$$

$r+5 \neq 0$
 $r \neq -5$

$r+10 \neq 0$
 $r \neq -10$

$$\frac{1}{(r+5)(r+10)}$$

$$\frac{5x^2 + 52x + 63}{5x + 45}$$

$5x+45 \neq 0$
 $5(x+9) \neq 0$
 $x \neq -9$

$$\frac{(5x+7)(x+9)}{5(x+9)}$$

$$\frac{5x+7}{5}$$

$$\frac{9a - 90}{5a^2 - 53a + 30}$$

~~$9(a-10)$~~
 ~~$(5a-3)(a-10)$~~

$5a-3 \neq 0$
 $+3 +3$
 $5a \neq 3$
 $a \neq \frac{3}{5}$

$a-10 \neq 0$
 $a \neq 10$

$$\frac{9}{(5a-3)}$$

$$\frac{3x^2 + 13x - 30}{6x^2 - 46x + 60}$$

$(6x-10)(x-6)$

$6x-10 \neq 0$
 $x \neq 10/6$

$(x-6) \neq 0$
 $x \neq 6$

$$\frac{(3x+5)(x-6)}{(6x-10)(x-6)}$$

$$\frac{(3x+5)}{(6x-10)}$$

Multiplying Rational Expressions: Multiply each of the following and state the excluded values.

$\frac{11p^2 \neq 0}{p \neq 0} \quad \frac{5 \rightarrow 16p^2}{11p^2 \rightarrow 3}$ $\frac{80p^2}{33p^2} = \frac{80}{33}$	$\frac{x-6 \neq 0}{x \neq 6} \quad \frac{x^2 - 4x - 12 \rightarrow x+7}{x-6} \cdot \frac{5x+10}{5x+10}$ $\frac{(x-6)(5x+10)}{(x-6)(5x+10)} = \frac{(x+7)}{5}$ $5x+10 \neq 0 \Rightarrow x \neq -2$
$n \neq -5, -8 \quad \frac{4n^2}{n+5} \cdot \frac{n^2+13n+40}{n+8}$ $\frac{4n^2(n^2+13n+40)}{(n+5)(n+8)}$ $\frac{4n^2(n+5)(n+8)}{(n+5)(n+8)} = 4n^2$	$\frac{3r+8}{3r^2+35r+72} \cdot \frac{5r(5r-2)}{5r^2+33r-14}$ $\frac{(3r+8)(r+9)(5r-2)(r+7)}{(3r+8)(r+9)(5r-2)(r+7)} = \frac{5r}{(r+9)(r+7)}$ $3r+8 \neq 0 \Rightarrow r \neq -8/3$ $r+9 \neq 0 \Rightarrow r \neq -9$ $5r-2 \neq 0 \Rightarrow r \neq 2/5$ $r+7 \neq 0 \Rightarrow r \neq -7$

Dividing Rational Expressions: Divide each of the following and state the excluded values.

$\frac{5a}{14} \div \frac{6}{4a}$ $\frac{5a}{14} \cdot \frac{4a}{6} = \frac{20a^2}{84} = \frac{5a^2}{21}$ $4a \neq 0 \Rightarrow a \neq 0$	$\frac{x-8}{x^2-2x-35} \div \frac{1}{x+5}$ $\frac{(x-8)(x+5)}{(x-7)(x+5)} \cdot \frac{1}{x+5} = \frac{(x-8)}{(x-7)(x+5)}$ $x-7 \neq 0 \Rightarrow x \neq 7$ $x+5 \neq 0 \Rightarrow x \neq -5$
$\frac{5a^2}{5a^3-50a^2} \div \frac{6a+14}{18a^3+42a^2}$ $\frac{(x+b)}{7}$ $x \neq -7/2, -4/7, -6, 0$	<p>3 fractions 3 denom.</p> $\frac{18x^3+63x^2}{14x+49} \cdot \frac{63x^3+36x^2}{7x^2+46x+24}$ $\frac{18x^3+63x^2}{14x+49} = \frac{9x^2(2x+7)}{7(2x+7)}$ $\frac{63x^3+36x^2}{7x^2+46x+24} = \frac{9x^2(x+6)}{7(x+4)(x+6)}$ $\frac{9x^2(2x+7)}{7(x+4)(x+6)} \cdot \frac{7(x+4)(x+6)}{9x^2(2x+7)} = \frac{9x^2(7x+4)}{(7x+4)(x+6)}$ $14x+49 \neq 0 \Rightarrow 7(2x+7) \neq 0 \Rightarrow x \neq -7/2$ $7x^2+46x+24 \neq 0 \Rightarrow (7x+4)(x+6) \neq 0 \Rightarrow x \neq -4/7, x \neq -6$ $63x^3+36x^2 \neq 0 \Rightarrow 9x^2(7x+4) \neq 0 \Rightarrow 9x^2 \neq 0 \Rightarrow x \neq 0$