Graphing Rational Functions

D OMAIN

R EDUCE

V ERTICAL ASYMPTOTES

H OLES

X -INTERCEPTS

Y -INTERCEPTS

E

G

ex:
$f(x) = \dfrac{(x+4)(x+1)}{(x+4)(x-2)}$

$x \neq -4, 2$

$D: (-\infty, -4) \cup (-4, 2) \cup (2, \infty)$

- all x-values from Left to Right
- Excluded values: are NOT included in the domain.
- Does NOT include vertical asymptotes, Holes

---

ex:
$f(x) = \dfrac{x^2 + 2x - 15}{x^2 + 8x + 15} = \dfrac{\cancel{(x+5)}(x-3)}{\cancel{(x+5)}(x+3)} = \dfrac{x-3}{x+3}$

- Factor the numerator and denominator
- cancel, if possible
- simplify

---

ex:
$f(x) = \dfrac{5}{(x-2)^3 (x+2)^4}$

V.A.: $x = 2$ odd
$x = -2$ even

- when the denominator is zero, the function is undefined.
- After reducing, set denom = 0 and solve for x.

ODD Multiplicity
ends go in opposite directions

EVEN multiplicity
ends go in SAME direction.

---
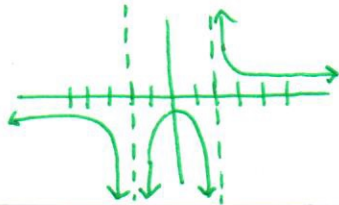
ex:
$f(x) = \dfrac{x^2 - 16}{x^2 + 9x + 20} = \dfrac{(x+4)(x-4)}{(x+4)(x+5)}$ hole @ x = -4
- Plug x value into what is "left"

$\dfrac{-4-4}{-4+5} = \dfrac{-8}{1} = -8$ Hole (-4, -8)

- Holes happen when we cancel factors COMPLETELY in numerator & denominator
ex: $\dfrac{(x+9)^2 (x+3)^3}{(x+3)^4 (x-3)}$ (x+3) doesn't cancel completely in denom., so x = -3 is NOT a hole, but V.A.

---

ex:
$f(x) = \dfrac{(x+2)^2 (x-3)}{(x+5)(x+4)}$

$x + 2 = 0$ $x - 3 = 0$
$x = -2$ $x = 3$
mult. 2   mult. 1
bounce   cross

- Plug 0 in for y and solve for x.
- The x-intercepts are the values of x that make the numerator equal zero.

★ Remember: even multiplicities: bounce
odd multiplicities: cross

---

ex:
$f(x) = \dfrac{2x^2 - 7x + 9}{4x^2 - 10x + 3} = \dfrac{2(0) - 7(0) + 9}{4(0) - 10(0) + 3} = \dfrac{9}{3} = 3$

$f(x) = \dfrac{(x-4)}{(x+4)(x+5)} = \dfrac{0-4}{(0+4)(0+5)} = \dfrac{-4}{(4)(5)} = \dfrac{-4}{20} = \dfrac{-1}{5}$

- Plug 0 in for x and solve for y.