

# Graphing Rational Functions

DOMAIN

REDUCE

VERTICAL ASYMPTOTES

HOLES

X-INTERCEPTS

Y-INTERCEPTS

END BEHAVIOR

GRAPH IT!

# Mrs. Adams, A3

ex:  
 $f(x) = \frac{(x+4)(x+1)}{(x+4)(x-2)}$   
 $x \neq -4, 2$

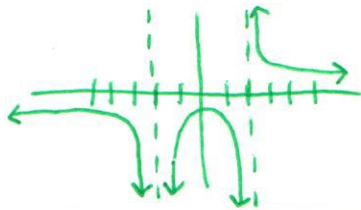
$D: (-\infty, -4) \cup (-4, 2) \cup (2, \infty)$

- all x-values from left to right
- excluded values are NOT part of domain
- Does NOT include V.A. and holes

ex:  
 $f(x) = \frac{x^2 + 2x - 15}{x^2 + 8x + 15} = \frac{\cancel{(x+5)}(x-3)}{\cancel{(x+5)}(x+3)} = \frac{x-3}{x+3}$

- Factor the numerator and denominator
- cancel, if possible
- simplify

ex:  
 $f(x) = \frac{5}{(x-2)^3(x+2)^4}$   
 VA:  $x=2$ , odd  
 $x=-2$ , even



- when the denominator is zero, the function is undefined.
- after reducing, set denom = 0 and solve for x.
- ODD Multiplicity ends go in opposite directions
- EVEN Multiplicity ends go in SAME direction

ex:  
 $f(x) = \frac{x^2 - 16}{x^2 + 9x + 20} = \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(x+5)}$  hole @  $x=-4$   
 - plug x-value of hole into what is "left"  
 $\frac{-4-4}{-4+5} = \frac{-8}{1} = -8$  Hole:  $(-4, -8)$

- holes happen when we cancel factors COMPLETELY in ~~numerator~~ denominator.

ex:  $f(x) = \frac{(x+9)^2 \cancel{(x+3)^3}}{(x+3)^4 (x-3)}$   $(x+3)$  doesn't cancel completely, so  $x=-3$  is NOT a hole, but V.A.

ex:  
 $f(x) = \frac{(x+2)^2(x-3)}{(x+5)(x+4)}$   
 $x=-2$  mult. 2 bounce  
 $x=3$  mult. 1 cross

- Plug 0 in for y and solve for x.
- The x-intercepts are the values of x that make the numerator equal zero.
- ★ Remember = odd multiplicities: cross  
even multiplicities: bounce

ex:  
 $f(x) = \frac{2x^2 - 7x + 9}{4x^2 - 10x + 3} = \frac{2(0) - 7(0) + 9}{4(0) - 10(0) + 3} = \frac{9}{3} = 3 = y$

- Plug 0 in for x and solve for y.

$f(x) = \frac{(x-4)}{(x+4)(x+5)} = \frac{(0-4)}{(0+4)(0+5)} = \frac{-4}{(4)(5)} = \frac{-4}{20} = \frac{-1}{5} = y$

ex:  
 $f(x) = \frac{x^2 + 4x + 3}{-4x^2 - 4x + 8}$   $m=2$ ,  $n=2$   
 $HA @ y = \frac{-1}{4}$   $m=n$   
 $y = \frac{a}{b}$

$f(x) = \frac{12x^5 + 9x^4 + 37x^3 + 9x + 1}{bx^3 + 4x^2 + 5x + 12}$   
 $m=5$ ,  $n=3$ ,  $m > n$  S.A.  
 $y = \frac{9x^m}{bx^n} = \frac{12x^5}{bx^3} = 2x^2 = y$

Given  $f(x) = \frac{ax^m + \dots}{bx^n + \dots}$  IF  
 $m > n$ , then  $y = \frac{ax^m}{bx^n}$  } slant asym.  
 $m = n$ , then  $y = \frac{a}{b}$  } horizontal asym.  
 $m < n$ , then  $y = 0$  }

★ only represents end behavior

YOU CAN DO IT! 😊

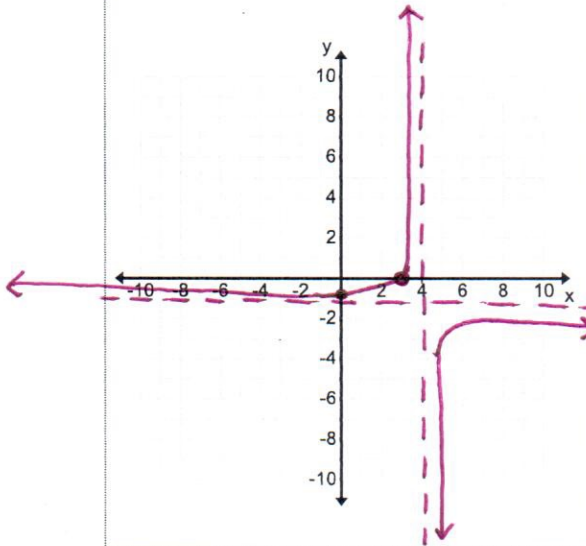
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Secondary 3 Honors: 4-2 Graphing Rational Functions Day 2 Notes

$$y = \frac{x-3}{-x+4}$$

$x \neq 4$

$m=1$     $n=1$     $m=n$     $y = \frac{a}{b}$   
 $y = \frac{1}{-1}$

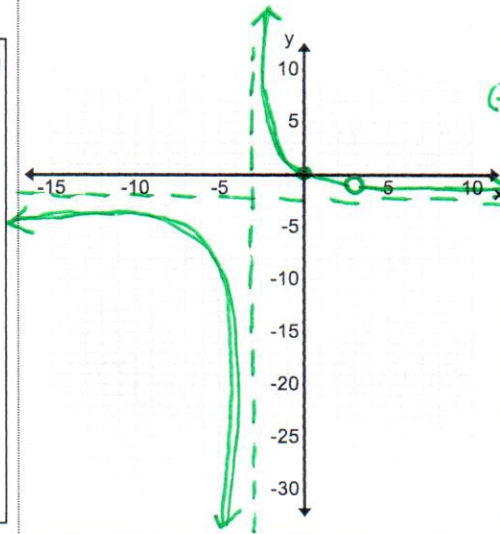


D:  $(-\infty, 4) \cup (4, \infty)$   
 VA:  $x=4$   
 H:  $y=-1$   
 X:  $x=3$  cross  
 Y:  $y=-3/4$   
 EB: HA @  $y=-1$

$$y = \frac{-2x(x-3)}{(x-3)(x+3)}$$

$x=3$   
 $\frac{-2(3)}{(3+3)} = \frac{-6}{6} = -1$

$m=2$     $n=2$     $m=n$     $y = \frac{a}{b}$   
 $y = \frac{-2}{1}$



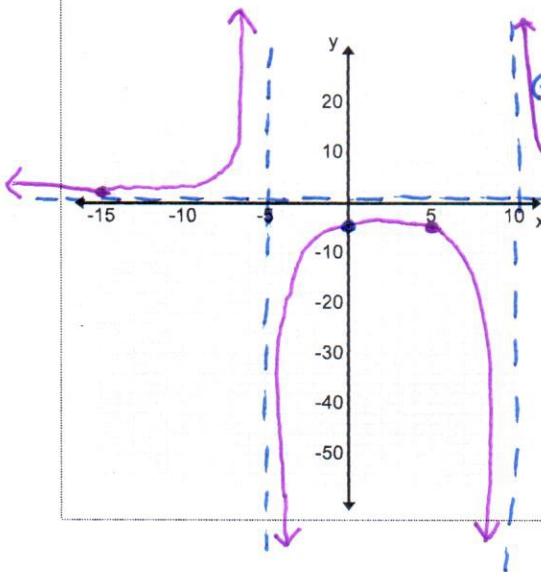
D:  $(-\infty, 3) \cup (3, \infty)$   
 VA:  $x=3$   
 H:  $y=-2$   
 X:  $x=0$  cross  
 Y:  $y=0$   
 EB: HA @  $y=-2$

$$y = \frac{1}{(x-2)(x+1)}$$

$\frac{1}{(0-2)(0+1)} = -\frac{1}{2}$

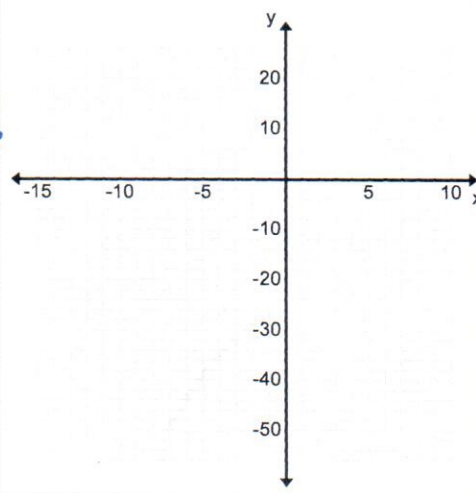
$m=0$     $n=2$     $m < n$     $y=0$

$x$	$f(x)$	$\{y\}$
1	-1/2	
-3	1/10	



D:  $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$   
 VA:  $x=-1$  opp  
 $x=2$  opp  
 H:  $y=-1/2$   
 X: NONE  
 Y:  $y=-1/2$   
 EB: HA @  $y=-1/2$

$$y = \frac{x^2(x-2)}{x(x-1)(x+2)}$$



D:  
 VA:  
 H:  
 X:  
 Y:  
 EB: