Name: Answer Key Class:

Unit 6: Exponentials and Logarithms Review

1. How do you know whether an exponential function has exponential growth or exponential decay?

   If the factor b is greater than 1 it grows

   If the factor b is between 0 and 1 it decays

2. Determine whether each function is an example of exponential growth or decay. Then find the y-intercept.

   a) $y = .17(0.92)^x$ $\boxed{\text{decay } (0, .17)}$

   b) $y = 240(1.8)^x$ $\boxed{\text{growth } (0, 240)}$

   c) $y = -\frac{1}{2}(7)^x$ $\boxed{\text{growth } (0, -\frac{1}{2})}$

Evaluate each logarithm

3. $\log_4 64$

   $4^x = 64$

   $4^x = 4^3$

   $\boxed{x=3}$

4. $\log_{125} 5$

   $125^x = 5$

   $(5^3)^x = 5^1$

   $\frac{3x}{3} = \frac{1}{3}$

   $\boxed{x = \frac{1}{3}}$

5. $\log_9 243$

   $9^x = 243$

   $(3^2)^x = 3^5$

   $2x = 5$

   $\boxed{x = \frac{5}{2}}$

Use the Change of Base Formula to evaluate the following expressions:

6. $\log_{15} 2$

   $\frac{\log 2}{\log 15} = \boxed{.25596}$

7. $\log_3 105$

   $\frac{\log 105}{\log 3} \approx \boxed{4.236217}$

Use the properties of logarithms to expand the following expressions:

8. $\log_3 \frac{9x}{y^2}$ $\boxed{(\log_3 9 + \log_3 x) - 2\log_3 y}$

   $\boxed{2 + \log_3 x - 2\log_3 y}$

9. $\log_6 18x^3y$

   $\log_6 18 + 3\log_6 x + \log_6 y$

   $\log_6 6 + \log_6 3 + 3\log_6 x + \log_6 y$

   $\boxed{1 + \log_6 3 + 3\log_6 x + \log_6 y}$

Use the properties of logarithms to simplify the following expressions:

10. $2\log_4 8 + \log_4 2 + \log_4 2$

    $\log_4 8^2 + \log_4 2 + \log_4 2$

    $\log_4 (8^2 \cdot 2 \cdot 2)$

    $\log_4 (256) = \boxed{4}$

11. $\log_3 4x + 2\log_3 5y$

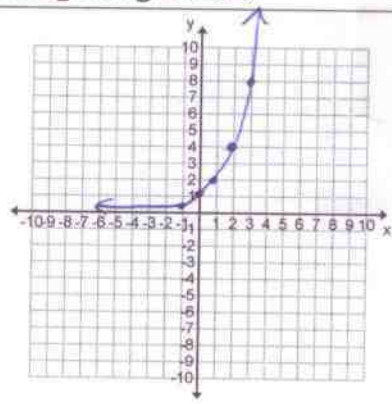
    $\log_3 4x + \log_3 (25y^2)$

    $\boxed{\log_3 (100xy^2)}$

Graph each of the following using a table.

12. $y = 2^x$

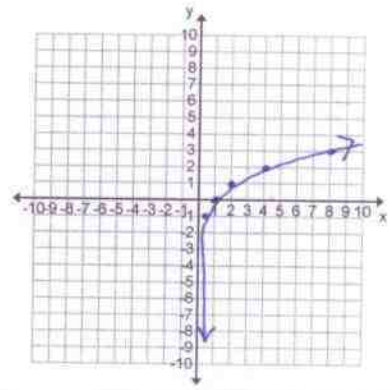
X	Y
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



13. $y = \log_2 x$

$2^y = x$

X	Y
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3



Use the properties of logarithms to solve the following logarithmic equations.

14. $2\log 5 + \log x = 2$

$\log 5^2 + \log x = 2$

$\log 25x = 2$

$10^2 = 25x$

$\frac{100}{25} = \frac{25x}{25}$

$\boxed{x = 4}$

15. $\log(x-21) + \log x = 2$

$\log(x^2 - 21x) = 2$

$10^2 = x^2 - 21x$

$100 = x^2 - 21x$

$x^2 - 21x - 100 = 0$

$(x-25)(x+4) = 0$

$\boxed{x = 25}$ $\cancel{x=-4}$

16. $\log 4x - \log 5 = 3$

$\log\left(\frac{4x}{5}\right) = 3$

$10^3 = \frac{4x}{5} \cdot 5$

$5 \cdot 1000 = 4x$

$\frac{5000}{4} = \frac{4x}{4}$

$\boxed{x = 1250}$

17. $\ln 3x = 8$

$\frac{e^8}{3} = \frac{3x}{3}$

$\boxed{x = 993.6526}$

18. $\ln(x-4) = 2$

$e^2 = x - 4$

$+4 \quad +4$

$e^2 + 4 = x$

$\boxed{x = 11.389}$

19. Describe how you would alter the equation $y = \log_b x$ to do the following transformations:

a. Translate (shift) vertically add or subtract to the end ex: $\log_b x + 4$
b. Translate (shift) horizontally add or subtract inside the log with x $y = \log_b(x+3)$
c. Reflect over the x-axis Multiply by a negative $y = -\log_b x$
d. Stretch Multiply by a number bigger than one $y = 4\log_b x$
e. Compress Multiply by a fraction smaller than one $y = \frac{1}{3}\log_b x$

20. A population of 752,000 decreases 1.4% per year. What size will the population be in 18 years.

$y = 752000(1 - .014)^x$

$y = 752000(.986)^{18}$

$\boxed{583{,}448.207}$

21. Sam invests \$5100 into an account with a 7% annual interest compounded continuously.

a. How long would it take to double his principal amount?

Pe^{rt}

$y = 5100e^{(.07)t}$

$10200 = 5100e^{(.07t)}$ $\quad 2 = e^{(.07t)}$ $\quad \frac{\ln 2}{.07} = \frac{.07t}{.07}$ $\quad \boxed{t = 9.9 \text{ years}}$

b. How long will it take for Sam's account balance to reach \$100,000?

$\frac{100{,}000}{5100} = \frac{5100}{5100}e^{(.07t)}$

$19.60784 = e^{.07t}$

$\frac{\ln(19.60784)}{.07} = \frac{.07t}{.07}$ $\quad \boxed{t = 42.513 \text{ years}}$

22. The apparent brightness of stars is measured on a logarithmic scale called magnitude, in which lower numbers means brighter stars. The relationship between the ratio of apparent brightness of two objects and the difference in their magnitudes is given by the formula $m_2 - m_1 = -2.5\log\frac{b_2}{b_1}$, where m is the magnitude and b is the apparent brightness. Compare the brightness of a magnitude 1.0 star with a magnitude 2.0 star.

$2.0 - 1.0 = -2.5\log\frac{b_2}{b_1}$

$\frac{1.0}{-2.5} = \frac{-2.5}{-2.5}\log\frac{b_2}{b_1}$

$-0.4 = \log\frac{b_2}{b_1}$

$10^{-0.4} = \frac{b_2}{b_1}$

$\frac{b_2}{b_1} = \boxed{.398107 \text{ times as bright}}$

– OR –

$1.0 - 2.0 = -2.5\log\frac{b_1}{b_2}$

$-1.0 = -2.5\log\frac{b_1}{b_2}$

$.4 = \log\frac{b_1}{b_2}$

$\boxed{2.5118 \text{ times as bright}}$

23. During a typical workday, the average sound intensity arriving at Larry's ears is 1.8×10^{-5}. Determine the loudness of these sounds he is hearing using the formula $L = 10\log\frac{I}{I_0}$, *where* $I_0 = 10^{-12}$.

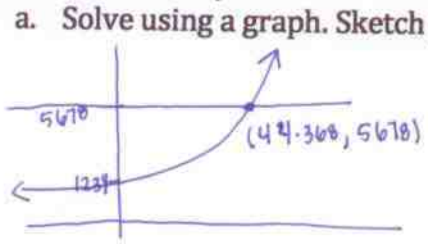
$10\log\frac{1.8\times10^{-5}}{10^{-12}} = \boxed{72.5527 \text{ decibels}}$

24. The population of an alien species named Spugawumbuoq increases at a rate of 3.5% per year. Currently their population is 1,234. At this rate, in how many years will there be 5,678 Spugawumbuoq ? Round your nearest answer to the hundredth (because Spugawumbuoq populations can have decimals.)

a. Solve using a graph. Sketch your graph below.

$y = 1234(1+.035)^x$

$y = 5678$



about $\boxed{44.37 \text{ years}}$

b. Solve algebraically. Make sure to show your work.

$\frac{5678}{1234} = \frac{1234(1.035)^x}{1234}$

$4.6012 = (1.035)^x$

$\log_{1.035} 4.6012 = x$

$x = 44.368$ years

about $\boxed{44.37 \text{ years}}$