REVIEW ANSWER KEY

Verify.

① $\boxed{\cot x \cos x + \sin x} = \csc x$

	complex side
$\frac{\cos x}{\sin x} \cdot \cos x + \sin x$	sines/cosines
	simplify
$\frac{\cos^2 x}{\sin x} + \frac{\sin x}{1} \cdot \frac{\sin x}{\sin x}$	common denominator
$\frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$	add
$\frac{\cos^2 x + \sin^2 x}{\sin x}$	pyth. identity
$\frac{1}{\sin x}$	Reciprocal identity
$\csc x$	$\blacksquare$

② $\boxed{\frac{\tan x}{\sec x + 1}} = \csc x - \cot x$

	complex side
$\frac{\tan x}{\sec x + 1} \cdot \frac{\sec x - 1}{\sec x - 1}$	conjugate
$\frac{\tan x(\sec x - 1)}{\sec^2 x - 1}$	pyth. identity
$\frac{\cancel{\tan x}(\sec x - 1)}{\tan^{\cancel{2}} x}$	simplify
$\frac{\sec x - 1}{\tan x}$	split fraction
$\frac{\sec x}{\tan x} - \frac{1}{\tan x}$	reciprocal identity
$\frac{\frac{1}{\cancel{\cos x}}}{\frac{\sin x}{\cancel{\cos x}}} - \cot x$	ratio identity; simplify
$\frac{1}{\sin x} - \cot x = \csc x - \cot x$	reciprocal identity $\blacksquare$

③ $\boxed{\frac{1-\cos x}{\sin x} + \frac{\sin x}{1-\cos x}} = 2\csc x$

	complex side
$\frac{(1-\cos x)}{(1-\cos x)} \frac{(1-\cos x)}{(\sin x)} + \frac{\sin x}{1-\cos x}\left(\frac{\sin x}{\sin x}\right)$	common denominator (single fraction)
$\frac{1 - 2\cos x + \cos^2 x}{(1-\cos x)(\sin x)} + \frac{\sin^2 x}{(1-\cos x)(\sin x)}$	add
$\frac{1 - 2\cos x + \cos^2 x + \sin^2 x}{(1-\cos x)(\sin x)}$	pyth. identity
$\frac{1 - 2\cos x + 1}{(1-\cos x)(\sin x)}$	combine like terms
$\frac{2 - 2\cos x}{(1-\cos x)(\sin x)}$	factor
$\frac{2\cancel{(1-\cos x)}}{\cancel{(1-\cos x)}(\sin x)}$	cancel
$\frac{2}{\sin x} = 2 \cdot \frac{1}{\sin x}$	reciprocal identity
$2\csc x$	$\blacksquare$

④ $\boxed{\tan x(\cot x+\tan x)}=\sec^2 x$

$\frac{\sin x}{\cos x}\left(\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}\right)$	complex side sines/cosines
$\frac{\cancel{\sin x\cos x}}{\cancel{\cos x\sin x}}+\frac{\sin^2 x}{\cos^2 x}$	Multiply
$1+\frac{\sin^2 x}{\cos^2 x}$	ratio identity
$1+\tan^2 x$	pyth. identity
$\sec^2 x$	∎

⑤ $\boxed{\sec x\sin x\cot x}=1$

$\frac{1}{\cos x}\cdot\frac{\sin x}{1}\cdot\frac{\cos x}{\sin x}$	complex side sines/cosines
$\frac{\cancel{\sin x}\cancel{\cos x}}{\cancel{\cos x}\cancel{\sin x}}$	Multiply simplify
1	∎

⑥ $\boxed{\cos x(\sec x-\cos x)}=\sin^2 x$

$\cos x\left(\frac{1}{\cos x}-\cos x\right)$	complex side sines/cosines
$\frac{\cos x}{\cos x}-\cos^2 x$	Multiply simplify
$1-\cos^2 x$	pyth. identity
$\sin^2 x$	∎

⑦ $\boxed{\sec^2 x-\tan^2 x}=1$

$\cancel{\tan^2}x+1-\cancel{\tan^2}x$	complex side pyth. identity
1	combine like terms ∎

⑧ $\boxed{\frac{\tan^2 x}{\tan^2 x+1}}=\sin^2 x$

$\frac{\tan^2 x}{\sec^2 x}$	complex side pyth. identity
$\frac{\frac{\sin^2 x}{\cancel{\cos^2 x}}}{\frac{1}{\cancel{\cos^2 x}}}$	sines/cosines simplify
$\sin^2 x$	∎

Find the exact value

⑨ $\cos 165°$

$\cos(120°+45°)$

$=\cos 120\cos 45-\sin 120\sin 45$

$=\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$=-\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}=\boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

⑩ $\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$=\sin\frac{\pi}{6}\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos\frac{\pi}{6}$

$=\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$=\frac{\sqrt{2}}{4}$

(11) $\sin 15° + \cos 75°$

$= \sin(45° - 30°) + \cos(45° + 30°)$

$= \sin 45° \cos 30° - \sin 30° \cos 45° +$
$\cos 45° \cos 30° - \sin 45° \sin 30°$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{2\sqrt{6}}{4} - \frac{2\sqrt{2}}{4} = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{2}}$

Write each of the following as a single trig expression.

(12) $\underset{\alpha}{\sin 8x} \underset{\beta}{\cos 2x} + \underset{\alpha}{\cos 8x} \underset{\beta}{\sin 2x}$

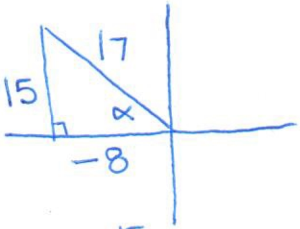
$= \sin(8x + 2x) = \boxed{\sin 10x}$

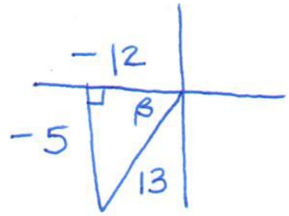
(13) $\underset{\alpha}{\cos 7x} \underset{\beta}{\cos x} + \underset{\alpha}{\sin 7x} \underset{\beta}{\sin x}$

$= \cos(7x - x) = \boxed{\cos 6x}$

Draw picture, then evaluate.

(14) Given $\cos\alpha = \frac{-8}{17}$ in Q2.
$\tan\beta = \frac{5}{12}$ in Q3.
Find $\sin(\alpha + \beta)$ and $\cos(\alpha - \beta)$.





$\sin\alpha = \frac{15}{17}$

$\cos\alpha = \frac{-8}{17}$

$\sin\beta = \frac{-5}{13}$

$\cos\beta = \frac{-12}{13}$

$\sin(\alpha + \beta)$

$= \sin\alpha \cos\beta + \sin\beta \cos\alpha$

$= \left(\frac{15}{17}\right)\left(\frac{-12}{13}\right) + \left(\frac{-5}{13}\right)\left(\frac{-8}{17}\right)$

$= \frac{-180}{221} + \frac{40}{221} = \boxed{\frac{-140}{221}}$

$\cos(\alpha - \beta)$

$= \cos\alpha \cos\beta + \sin\alpha \sin\beta$

$= \left(\frac{-8}{17}\right)\left(\frac{-12}{13}\right) + \left(\frac{15}{17}\right)\left(\frac{-5}{13}\right)$

$= \frac{96}{221} + \frac{-75}{221} = \boxed{\frac{21}{221}}$

Verify the following.

(15) $\boxed{\sin\left(\underset{\alpha}{x} - \underset{\beta}{\frac{\pi}{4}}\right)} = \frac{\sqrt{2}}{2}(\sin x - \cos x)$

	complex side
$\sin x \cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos x$	sum/diff identity
$\sin x\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\cos x$	unit circle
$\frac{\sqrt{2}}{2}(\sin x - \cos x)$	FACTOR
	■

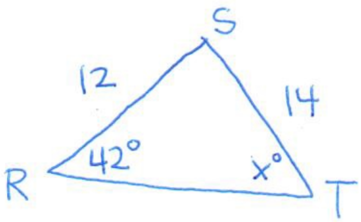
(16) $\boxed{\cos(x + \pi)} = -\cos x$

	complex side
$\cos x \cos\pi - \sin x \sin\pi$	sum/diff. identity
$\cos x(-1) \cancel{- \sin x(0)}$	unit circle
$-\cos x$	simplify
	■

Use the Law of Sines to solve. Round answers to nearest hundredth.

(17) In $\triangle RST$, $m\angle R = 42°$, $t = 12$ in, $r = 14$ in. Find $m\angle T$.

SSA $\triangle$



$$\frac{\sin 42°}{14} = \frac{\sin x°}{12}$$

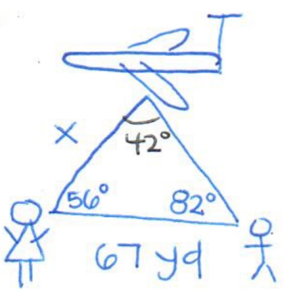
$$\frac{12\sin 42°}{14} = \frac{\cancel{14}\sin x°}{\cancel{14}}$$

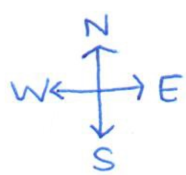
$$\cancel{\sin^{-1}}(\sin x°) = \sin^{-1}(0.57354)$$

$$\boxed{x° = 35.00°}$$

(18) 2 people walking, 1 plane overhead. How far is the plane from the female? Draw a picture.

ASA $\triangle$





$180 - 56 - 82 = 42°$

$$\frac{\sin 42°}{67} = \frac{\sin 82°}{x}$$

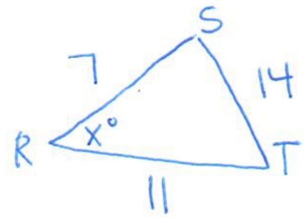
$$\frac{x\cancel{\sin 42°}}{\cancel{\sin 42°}} = \frac{67\sin 82°}{\sin 42°}$$

$$\boxed{x = 99.16 \text{ yd}}$$

Use the Law of Cosines to solve. Round answers to nearest hundredth.

(19) In $\triangle RST$, $r = 14$ cm, $s = 11$ cm, $t = 7$ cm. Find $m\angle R$.

SSS $\triangle$



$$14^2 = 7^2 + 11^2 - 2(7)(11)\cos x°$$

$$196 = \cancel{170} - 154\cos x°$$

$$-170 \quad \cancel{-170}$$

$$\frac{26}{-154} = \frac{\cancel{-154}\cos x°}{\cancel{-154}}$$

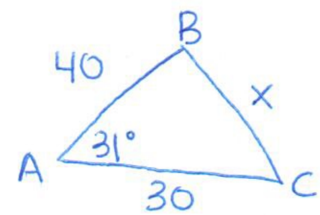
$$-0.1688 = \cos x°$$

$$x° = \cos^{-1}(-0.1688)$$

$$\boxed{x° = 99.72°}$$

(20) In $\triangle ABC$, $A = 31°$, $b = 30$mm, $c = 40$ mm. Find a.

SAS $\triangle$



$$x^2 = 30^2 + 40^2 - 2(30)(40)\cos 31°$$

$$x^2 = 2500 - 2057.202$$

$$x^2 = 442.798$$

$$\boxed{x = 21.04 \text{mm}}$$

21) use the Law of Sines to Find m∠B in #20.

$$\frac{\sin 31^\circ}{21.04} = \frac{\sin B}{30}$$

$$\frac{30\sin 31^\circ}{21.04} = \frac{\cancel{21.04}\sin B}{\cancel{21.04}}$$

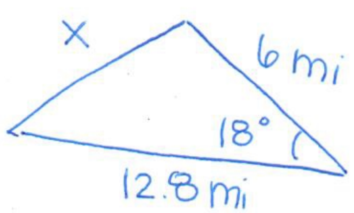
$$\sin B = 0.734$$

$$B = \sin^{-1}(0.734)$$

$$\boxed{B = 47.25^\circ}$$

22) You are going on a trip. You encounter a roadblock and must go around. How many extra miles will you have to travel? Then find Area.

SAS△



$$x^2 = (12.8)^2 + (6)^2 - 2(6)(12.8)\cos 18^\circ$$

$$x^2 = 199.84 - 146.08$$

$$x^2 = 53.76$$

$$x = 7.33 \text{ miles}$$

EXTRA MILES

$$[6 + 7.33] - 12.8$$

$$= \boxed{0.53 \text{ more miles}}$$

$$A_\triangle = \frac{1}{2}(6)(12.8)\sin 18^\circ$$

$$\boxed{A_\triangle = 11.87 \text{ mi}^2}$$