Secondary Math 3 Honors Name: key

Sage Review Homework Packet

Higher Order Polynomials

Describe the end behavior using limits.

1. $f(x) = -x^5 + 4x^3 + 7x + 2$

odd, negative

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = \infty$

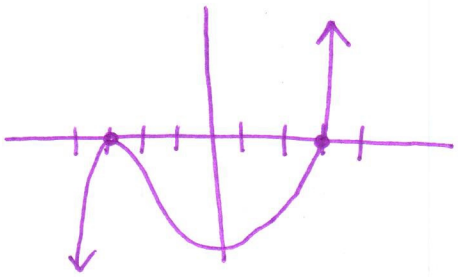
2. State the degree, list zeros, state multiplicity, what happens at that zero and then sketch the graph of the polynomial by hand. $f(x) = (x+3)^2(x-3)^5$

Degree: 7 +, odd

Zeroes: $x = -3$ $x = 3$

mult. 2 mult. 5

Bounce cross



Use the Remainder Theorem to find the remainder when f(x) is divided by $x - k$.

3. $f(x) = 7x^4 + 5x^3 + 2x^2 - 3x + 3 \quad k = 2$

$r = f(k) = f(2)$

$f(2) = 7(2)^4 + 5(2)^3 + 2(2)^2 - 3(2) + 3$

$\boxed{r = 157}$

Factor each of the following:

4. $7m^2 - 50m - 48$ ac

$= 7m^2 + 6m - 56m - 48$ -336

$= m(7m+6) - 8(7m+6)$ $-12, 28$

$= \boxed{(7m+6)(m-8)}$ $-16, 21$

$-6, 56$

$\boxed{6, -56}$

5. $x^3 + 125$ $a^3 + b^3$

$a = x \quad b = 5$

$\boxed{(x+5)(x^2 - 5x + 25)}$

$(a+b)(a^2 - ab + b^2)$

6. $10x^3 + 15x^2 + 12x + 18$

$= 5x^2(2x+3) + 6(2x+3)$

$= \boxed{(2x+3)(5x^2+6)}$

Sequences and Series

Write the next 3 terms of the pattern. Then write the explicit and recursive rule for the following arithmetic or geometric sequences:

1. $10, 14, 18, 22, \ldots$ arithmetic

   $+4 \quad +4$

   Next 3 terms: $26, 30, 34$

   Recursive: $a_n = a_{n-1} + 4$ $\quad a_n = a_{n-1} + d$

   Explicit: $a_n = 10 + (n-1)(4)$ $\quad a_n = 4n + 6$ $\quad a_n = a_1 + (n-1)d$

2. $12, 6, 3, 1.5, \ldots$ geometric

   $\times 1/2 \quad \times 1/2$

   Next 3 terms: $0.75, 0.375, 0.1875$

   Recursive: $a_n = \frac{a_{n-1}}{2}$ $\quad a_n = a_{n-1} \cdot r$

   Explicit: $a_n = 12\left(\frac{1}{2}\right)^{n-1}$ $\quad a_n = a_1 \cdot r^{n-1}$

Find the sum of the arithmetic or geometric sequence.

3. Find the sum of the sequence of the first 20 terms: $a_n = 3n + 1$

   $a_1 = 4$

   $a_{20} = 61$

   $S_n = \frac{n}{2}(a_1 + a_n)$ $\quad S_{20} = \frac{20}{2}(4 + 61)$

   $S_{20} = 650$

4. Find the sum of the sequence of the first 30 terms.

   $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$

   $r = \frac{1}{2} < 1$ converges

   $a_1 = 1/4$

   $n = 30$

   $S_n = \frac{a_1(1-r^n)}{1-r}$

   $S_{30} = \frac{\frac{1}{4}\left(1-\left(\frac{1}{2}\right)^{30}\right)}{1-1/2} = 0.5$

Determine if the following series converges or diverges. If it converges find the sum.

5. $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)\left(\frac{1}{3}\right)^n$ $\quad a_1 = \left(\frac{3}{4}\right)\left(\frac{1}{3}\right)^1 = \frac{1}{4}$

   $r = \frac{1}{3} < 1$ converges

   $S = \frac{a_1}{1-r} = \frac{1/4}{1-1/3} = \frac{3}{8}$

6. $1 + 3 + 9 + 27 + \ldots$

   diverges

Rational Expressions

Simplify each and state the excluded values.

1. $\frac{18a^5}{54a} = \frac{1}{3}\frac{18aaaaa}{54a} = \frac{1a^4}{3}$ $\quad a \neq 0$

2. $\frac{x^2-3x-10}{x+2} = \frac{(x-5)(x+2)}{x+2} = x-5$

   $x \neq -2$

3. $\frac{5}{2r-10} \div \frac{7r^2+37r-30}{14r-10}$

   $\frac{5}{2(r-5)} \times \frac{2(7r-5)}{(7r-5)(r+6)} = \frac{5}{(r-5)(r+6)}$

   $r \neq 5, 5/7, -6$

4. $\frac{5}{n-3} - \frac{3}{n-2}$ common denominator $(n-3)(n-2)$

   $\frac{(n-2)}{(n-2)}\left(\frac{5}{n-3}\right) - \left[\frac{(n-3)}{(n-3)}\left(\frac{3}{n-2}\right)\right]$

   $= \frac{5n-10-3n+9}{(n-2)(n-3)} = \frac{2n-1}{(n-2)(n-3)}$

   $n \neq 3, 2$

5. $\frac{\frac{9}{x+1} + \frac{x+1}{9}}{\frac{x+1}{3}}$ common denom: $9(x+1)$

   $\frac{\frac{(9)}{(9)}\left(\frac{9}{x+1}\right) + \frac{(x+1)}{(x+1)}\left(\frac{x+1}{9}\right)}{\frac{(3(x+1))}{(3(x+1))}\left(\frac{x+1}{3}\right)} = \frac{\frac{81+(x+1)^2}{9(x+1)}}{\frac{3(x+1)^2}{9(x+1)}} = \frac{81+x^2+2x+1}{3(x^2+2x+1)} = \frac{x^2+2x+82}{3x^2+6x+3}$

   $x \neq -1$

Rational Functions

1. Find all of the critical information for the following functions and graph:

$$f(x) = \frac{x^2 - 36}{x + 4} = \frac{(x+6)(x-6)}{(x+4)}$$

D: $(-\infty, -4) \cup (-4, \infty)$

V: $x = -4$ odd, opp

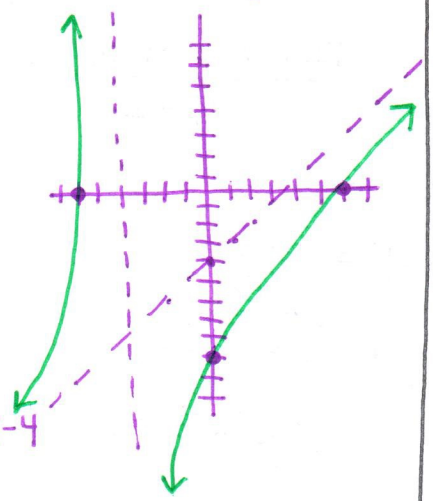
H: No holes

X: $x = 6$, $x = -6$
odd odd
cross

Y: $\frac{-36}{4}$ $y = -9$

E: $m = 2$ $n = 1$ $m > n$ so SA @ $y = x - 4$

$$\begin{array}{r} x-4 \\ x+4 \overline{)\,x^2+0x-36} \\ -(x^2+4x) \\ \hline -4x-36 \\ -(-4x+16) \\ \hline -20 \text{ Rem.} \end{array}$$



$$f(x) = \frac{x^2 - x - 12}{x^2 + x - 2} = \frac{(x-4)(x+3)}{(x+2)(x-1)}$$

D: $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

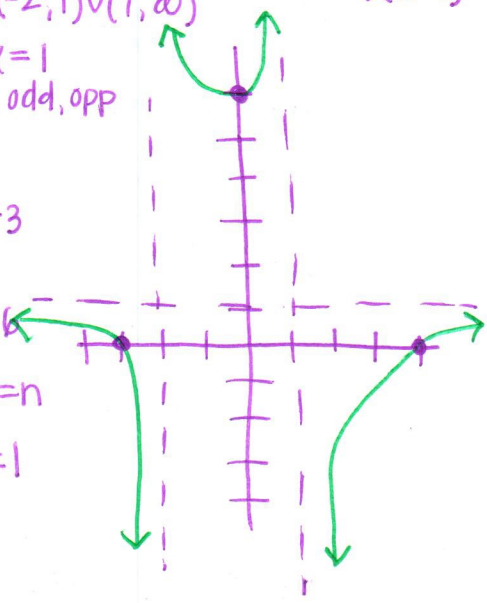
V: $x = -2$, $x = 1$
odd, opp odd, opp

H: NONE

X: $x = 4$, $x = -3$
cross

Y: $\frac{-12}{-2}$ $y = 6$

E: $m = 2$ $n = 2$ $m = n$
HA @ $y = 1$



2. Solve the following rational function:

Use lcd!

lcd = $x(x+4)$

$$\frac{2}{1} - \frac{3}{x+4} = \frac{12}{x^2 + 4x}$$

$$x(x+4)\left(\frac{2}{1}\right) - x(x+4)\left(\frac{3}{x+4}\right) = x(x+4)\left(\frac{12}{x(x+4)}\right)$$

$2x^2 + 8x - 3x = 12$

$2x^2 + 5x - 12 = 0$

$(2x-3)(x+4) = 0$

$x = 3/2$

$x = -4$ ← ex. sol.

3. Miquel can complete the decorations for a school dance in 5 days working alone. Nasim can do it alone in 3 days, and Denise can do it alone in 4 days. How long would it take the three students working together to decroate for a school dance?

$$\frac{1}{5} + \frac{1}{3} + \frac{1}{4} = \frac{1}{x}$$

common denom = $60x$

$$\frac{12x}{12x}\left(\frac{1}{5}\right) + \frac{20x}{20x}\left(\frac{1}{3}\right) + \frac{15x}{15x}\left(\frac{1}{4}\right) = \frac{60}{60}\left(\frac{1}{x}\right)$$

$$\frac{12x}{60x} + \frac{20x}{60x} + \frac{15x}{60x} = \frac{60}{60x}$$

$$(60x)\left(\frac{47x}{60x}\right) = \left(\frac{60}{60x}\right)(60x)$$

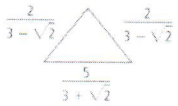
$47x = 60$

$x = \frac{60}{47}$

$x \approx 1.28$ days

Radicals

1. What is the perimeter of the triangle at the right?



$P = \frac{2}{3-\sqrt{2}} + \frac{2}{3-\sqrt{2}} + \frac{5}{3+\sqrt{2}}$

$= \frac{4}{3-\sqrt{2}} + \frac{5}{3+\sqrt{2}}$

common denom $(3-\sqrt{2})(3+\sqrt{2})$ $= 9-2$

$\frac{(3+\sqrt{2})}{(3+\sqrt{2})}\left(\frac{4}{3-\sqrt{2}}\right) + \frac{(3-\sqrt{2})}{(3-\sqrt{2})}\left(\frac{5}{3+\sqrt{2}}\right)$

$\frac{12+4\sqrt{2}+15-5\sqrt{2}}{9-2} = \boxed{\frac{27-\sqrt{2}}{7}}$

2. A spherical water tank holds 6000 ft³ of water. What is the diameter of the tank to the nearest tenth of a foot? (Hint: $V = \frac{\pi}{6}d^3$)

$V = 6000$

$6000 = \frac{\pi}{6}d^3$

$d^3 = \frac{6}{\pi}(6000)$

$d^3 = \frac{36000}{\pi}$

$d = \sqrt[3]{\frac{36000}{\pi}}$

$\boxed{d = 22.5 \text{ ft}}$

3. The area of a triangle is 14 in^2. The height is $(4+\sqrt{3})in$. What is the width?

$A = \frac{1}{2}wh$

$14 = \frac{1}{2}w(4+\sqrt{3})$ $\quad 28 = (4+\sqrt{3})w$

$w = \frac{28}{4+\sqrt{3}}$ Rationalize the denom!

$w = \frac{28}{4+\sqrt{3}} \cdot \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{112-28\sqrt{3}}{16-3} = \boxed{\frac{112-28\sqrt{3}}{13} \text{ in}}$

Exponentials and Logs

1. A new boat that sells for $16500 depreciates 18% each year. Write an exponential function to model the situation and then find the amount of the boat after 6 years.

$a_1 = 16500$
$r = -0.18$
$t = 6$

$A = 16500(1-0.18)^6$

$= \boxed{\$5016.11}$

2. Sam invests $5100 into an account with a 7% annual interest compounded continuously. $y = Pe^{rt}$
   - How long would it take to double his principal amount?

$P = 5100$
$2P = 10200$
$r = 0.07$

$10200 = 5100e^{(0.07)t}$

$2 = e^{0.07t}$

$\ln 2 = 0.07t$ $\quad t = \frac{\ln 2}{0.07} = \boxed{9.9 \text{ yrs}}$

   - How long will it take for Sam's account balance to reach $100,000?

$100000 = 5100e^{0.07t}$

$19.6078 = e^{0.07t}$

$\ln 19.6078 = 0.07t$ $\quad \frac{\ln 19.6078}{0.07}$ $\quad \boxed{t = 42.5 \text{ yrs}}$

3. A parent increases a child's allowance by 15% each year. If he allowance is $3 now, how much will it be in 5 years?

$a_1 = 3$
$r = 0.15$
$t = 5$

$A = 3(1+0.15)^5$

$= \boxed{\$6.03}$

4. Solve the following equation for x: $\ln(x-4) = 2$

$\ln(x-4) = 2$

$e^2 = x-4$
$+4 \quad +4$

$\boxed{x = e^2+4}$ or $\boxed{x = 11.389}$

GREEN

Trigonometry **Name:**_____

1. At the Delaware State Fair, there is a Ferris wheel with a 24 meter diameter. The center of the Ferris wheel is 15 meters off the ground. The Ferris wheel completes one rotation in 48 seconds.

period=48 $\frac{2\pi}{b}=48$ $b=\frac{\pi}{24}$

a) Write an equation to model the height of a rider on this Ferris wheel h at any time t, where $t = 0$ is when the rider is at the position on the Ferris wheel farthest to the right.

$y=12\sin\left(\frac{\pi}{24}x\right)+15$

b) Revise your equation so that $t = 0$ when the rider is at the bottom of the Ferris wheel.

$y=12\sin\frac{\pi}{24}(x-12)+15$

c) What is the height of the rider at 42 seconds?

$y=12\sin\left(\frac{\pi}{24}(42)\right)+15$ $y=6.51$ meters

2. On tractors, rolling circumference can be defined as the distance a tire travels in one revolution. Since both the front and the rear tire on tractors are engaged, the front tire requires more revolutions to cover the same distance as the rear tire. On one particular tractor, the front tire has a 18-inch radius and the rear tire has a 48-inch radius.

a) Find the circumference of each tire.

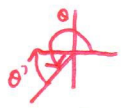
$C=2\pi r$ Front: $2\pi(18)=36\pi$ in Back: $2\pi(48)=96\pi$ in

b) If the tractor travels 100 miles, how many times does each tire rotate? (Remember that 1 mile = 63360 inches.)

Front: $\frac{100\text{mi}}{1}\cdot\frac{63360\text{ in}}{1\text{mi}}\cdot\frac{1}{36\pi\text{ in}}=56022.54$ rot

Back: $\frac{100\text{mi}}{1}\cdot\frac{63360\text{ in}}{1\text{mi}}\cdot\frac{1}{96\pi\text{in}}=21008.45$ rot

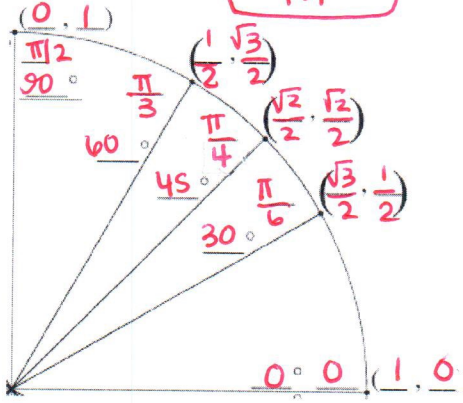
3. Fill in the missing values on the unit circle **AND** then evaluate the two functions.



a. $\cos\frac{7\pi}{6}$ Q3: − $\theta'=\frac{\pi}{6}$ $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ $\cos\frac{7\pi}{6}=-\frac{\sqrt{3}}{2}$

b. $\sin\frac{2\pi}{3}$ $\theta'=\frac{\pi}{3}$ Q2: + $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$ $\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$

c. $\cos\frac{5\pi}{4}$ $\theta=\frac{\pi}{4}$ $\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$ Q3: − $\cos\frac{5\pi}{4}=-\frac{\sqrt{2}}{2}$



4. Convert the following from degrees to radians or from radians to degrees.

a. $200^\circ\cdot\frac{\pi}{180^\circ}=\frac{10\pi}{9}$

b. $415^\circ\cdot\frac{\pi}{180^\circ}=\frac{83\pi}{36}$

c. $\frac{6\pi}{4}\cdot\frac{180^\circ}{\pi}=270^\circ$

d. $\frac{5\pi}{3}\cdot\frac{180^\circ}{\pi}=300^\circ$

Statistics

Great Lakes Coastal Water Temperatures (°F)												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Buffalo, NY	34	28	33	46	56	65	72	70	61	50	44	38
Oswego, NY	49	48	48	49	52	57	62	65	64	62	58	54

1. Find the mean and the standard deviation of the water temperature for Buffalo, NY and Oswego, NY.

Buffalo
$\bar{x} = 49.75$
$\sigma = 14.41$

Oswego
$\bar{x} = 55.67$
$\sigma = 6.24$

2. To investigate a community's reading habits, a newspaper conducts a poll from a table near the exit of a history museum.

a) What is the sampling method? convenience

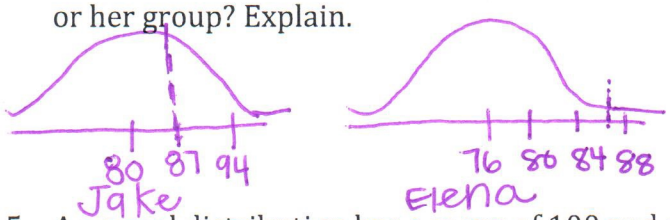
b) Does the sampling method have any bias? Explain.

Yes, explain in your own words

3. What does it mean to have an unbiased sample? Why does it matter?

- Simple Random Sample
- Everyone has an equal chance
- Results are not skewed
- conclusions more accurate

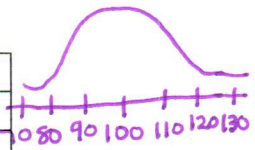
4. Jake and Elena took the same standardized test, but are in different classes. They both received a score of 87. In Jake's group, the mean was 80 and the standard deviation was 7. In Elena's group, the mean was 76 and the standard deviation was 4. Did either student score in the top 2.5% of his or her group? Explain.



Elena did because 87 is between 2 and 3 std. dev. above mean.

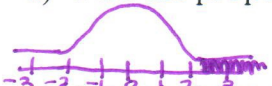
5. A normal distribution has a mean of 100 and a standard deviation of 10. Find the probability that a value selected at random is in the given interval. 68 - 95 - 99.7

From 80 to 110	81.5%	From 70 to 130	99.7%
From 90 to 120	81.5%	At least 100	50%
At most 110	84%	At least 80	97.5%



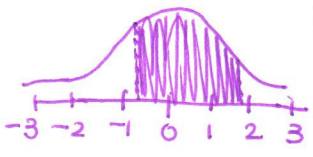
For a standard normal curve, draw and shade in the given region. Then solve for the proportions under a standard normal curve. use z-score table

6. a) Find the proportion of observations that fall into the region. $z > 2.17$



1.00 − .9850 = 0.015 → 1.5%

b) Find the proportion of observations that fall into the region. $-0.66 < z < 1.95$



prob. for 1.95 = 0.9744
prob. for −0.66 = 0.2546
= 0.9744 − 0.2546
= 0.7198 so 72%

Trig Identities

1. Verify the following expression: $\cot x + \tan x = \sec x \csc x$

$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$\frac{\cos x}{\cos x}\left(\frac{\cos x}{\sin x}\right) + \frac{\sin x}{\sin x}\left(\frac{\sin x}{\cos x}\right)$

$\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$

$\frac{1}{\sin x \cos x}$

difficult sines/cosines

common denom.

multiply and add

pyth. identity

$\frac{1}{\sin x} \cdot \frac{1}{\cos x}$

$\csc x \cdot \sec x$

Split fraction

Reciprocal Identities

■

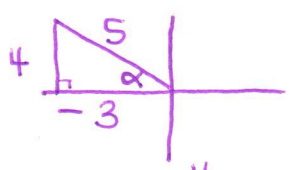
2. Given $\tan \alpha = -\frac{4}{3}$ in Quadrant II, and $\tan \beta = \frac{15}{8}$ in Quadrant III, find the following:

a. $\sin(\alpha - \beta)$

$= \sin\alpha\cos\beta - \sin\beta\cos\alpha$

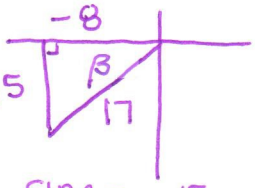
$= \left(\frac{4}{5}\right)\left(\frac{-8}{17}\right) - \left(\frac{-15}{17}\right)\left(\frac{-3}{5}\right)$

$= \frac{-32}{85} - \frac{45}{85} = \boxed{\frac{-77}{85}}$



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{-3}{5}$



$\sin\beta = \frac{-15}{17}$

$\cos\beta = \frac{-8}{17}$

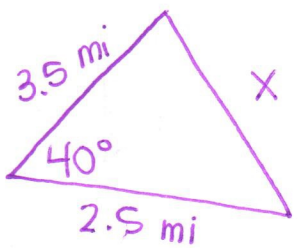
b. $\cos(\alpha + \beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{-3}{5}\right)\left(\frac{-8}{17}\right) - \left(\frac{4}{5}\right)\left(\frac{-15}{17}\right)$

$= \frac{24}{85} - \frac{-60}{85} = \boxed{\frac{84}{85}}$

3. The sailboat race committee wants to lay out a triangular course with a 40° angle between two sides that measure 3.5 miles and 2.5 miles. What will be the approximate length of the third side?



SAS Δ → use Law of Cosines

$x^2 = (3.5)^2 + (2.5)^2 - 2(3.5)(2.5)\cos 40°$

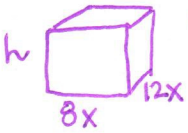
$x^2 = 5.09$

$\boxed{x = 2.26 \text{ mi}}$

Geometry

DIMENSIONS: 18.8 in × 28.2 in × 22.6 in

1. A company wants to manufacture packaging boxes in the shape of rectangular prisms. Each box will have a volume of 12,000 cubic inches. The company wants to choose the dimensions of a box with side lengths h in, 8x in., and 12x in., so that the box's surface area is minimized. What dimensions should the company choose for the boxes? Round your answer to the nearest tenth.



$V = 12000$

$(8x)(12x)(h) = 12000$

$96x^2h = 12000$

$h = \frac{125}{x^2}$

$SA = 2(h)(8x) + 2(8x)(12x) + 2(h)(12x)$

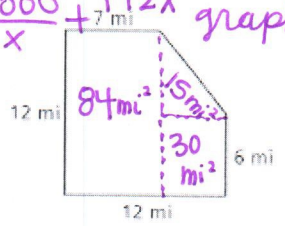
$= 16xh + 192x^2 + 24xh$

$= 40xh + 192x^2$

$= 40x\left(\frac{125}{x^2}\right) + 192x^2 = \frac{5000}{x} + 192x^2$

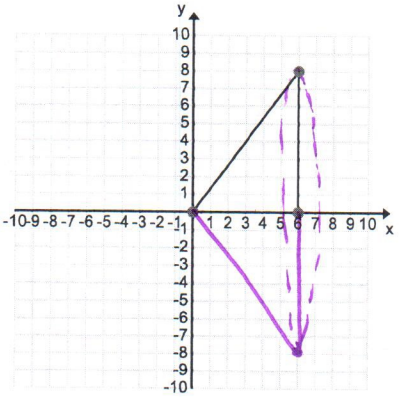
min (2.35, 3187.9) graph!

The population of a nearby county is 92,024. The dimensions of the county are shown at the right. What is the population density?



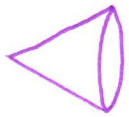
$PD = \frac{pop}{area} = \frac{92024}{129} =$ 713 people/mi²

Suppose the right triangle shown below is rotating rapidly about the x-axis. Like a spinning skater, a solid image would be formed by the blur of the rotating triangle.



2. Draw and describe the solid of revolution formed by rotating this triangle about the x-axis.

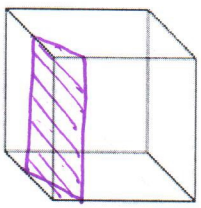
cone.



3. Find the volume of the solid formed.

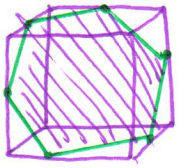
$V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)^2(6) =$ 128π units³ or 402.12 units³

4. Draw and describe a cross section formed by a plane intersecting the cube as follows.

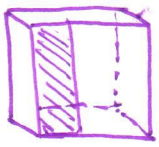


a. The plane is tilted and intersects the left and right faces of the cube.

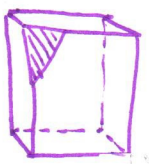
Rectangle or hexagon



b. The plane cuts off a corner of the cube.



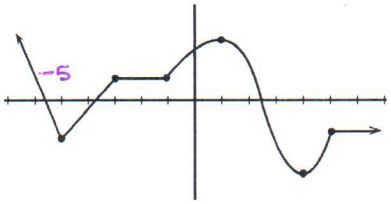
Rectangle



Triangle

Functions

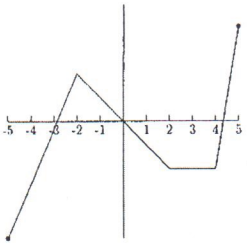
Determine the open intervals on which each function is increasing, decreasing or constant. Write your answers in interval notation.



Increasing: $(-5,-3), (-1,1), (4,5)$

Decreasing: $(-\infty,-5)\ (1,4)$

Constant: $(-3,-1)\ (5,\infty)$



Increasing: $(-\infty,-2),\ (4,\infty)$

Decreasing: $(-2,2)$

Constant: $(2,4)$

Use your calculator to find the exact point (to 3 decimal places) of any relative extrema of the following:

$f(x) = x^3 + 6x^2 - 12$

Max: $(-4,20)$
Min: $(0,-12)$

$f(x) = x\sqrt{5+x}$

Min: $(-3.333,-4.303)$

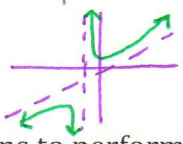
$f(x) = x^2 - 12x + 90$

min: $(6,54)$

Find the left-hand and right-hand limits of the functions, using limit notation, as x approaches -1.

$f(x) = \dfrac{x^2 - 2x + 3}{x + 1}$

Graph it!



$\lim_{x \to -1^-} f(x) = -\infty$

$\lim_{x \to -1^+} f(x) = \infty$

Use the following functions to perform the following operations:

$f(x) = x^3 \quad g(x) = 2x + 7 \quad h(x) = x + 2 \quad j(x) = 4 - x^2 \quad k(x) = \sqrt{x^2 + 4} \quad m(x) = x^3 + 1$

1. $f(g(x))$

$= (2x+7)^3$

$= 8x^3 + 84x^2 + 294x + 343$

2. $g(f(x))$

$2(x^3) + 7$

$2x^3 + 7$

3. $f(f(x))$

$(x^3)^3$

x^9

4. $g(g(x))$

$2(2x+7) + 7$

$4x + 14 + 7$

$4x + 21$

5. $h(j(x))$

$(4 - x^2) + 2$

$-x^2 + 6$

6. $j(h(x))$

$4 - (x+2)^2$

$4 - (x^2 + 4x + 4)$

$-x^2 - 4x$

7. $k(m(x))$

$\sqrt{(x^3+1)^2 + 4}$

$\sqrt{x^6 + 2x^3 + 1 + 4}$

$\sqrt{x^6 + 2x^3 + 5}$

8. $j(f(x))$

$4 - (x^3)^2$

$4 - x^6$