Name: Key Period: ________

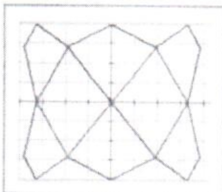
Unit 11 Review

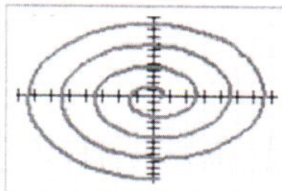
SM3H | Polar and Parametric Equations | Unit 11

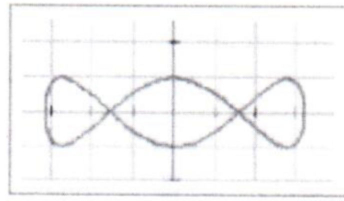
Match the parametric equations with their graphs.

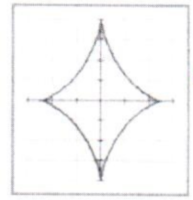
1. $x = 6\sin(4t)$ and $y = 4\sin(6t)$

2. $x = 4(\cos t)^3$ and $y = 4(\sin t)^3$





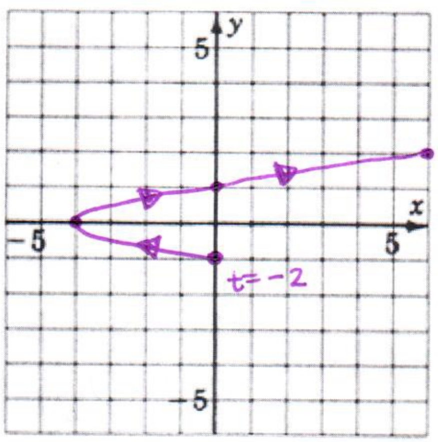




Sketch the curve given by the parametric equations.

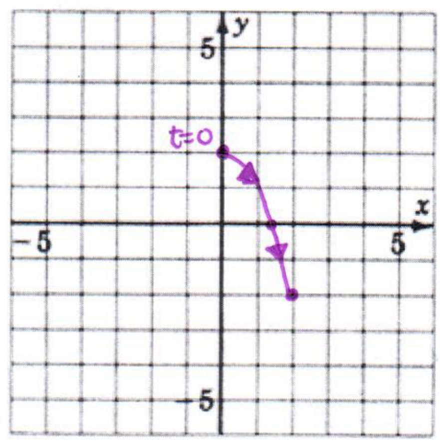
3. $x = t^2 - 4$ and $y = \dfrac{t}{2}$

(Set Tmin: -2 and Tmax: 3)



4. $x = \sqrt{t}$ and $y = 2 - t$

(Set Tmin: 0 and Tmax: 4)

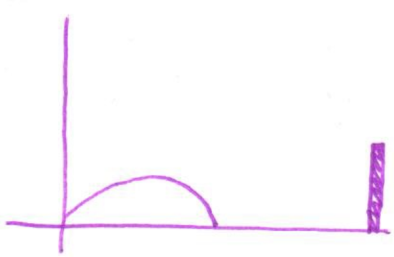


5. Bryce Harper hits a baseball 3 feet above the ground, with an initial velocity of 100 ft/sec at an angle of 15° with the horizontal. Using the following parametric equations, will the ball clear a 10 foot wall that is 400 feet away? Draw a picture to help you answer the question.

DEGREE MODE

$$x = (100\cos 15)t \quad \text{and} \quad y = -16t^2 + (100\sin 15)t + 3$$

$x_2 = 400$ $y_2 = 10$ to graph the fence

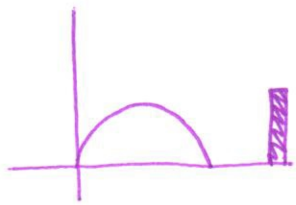


Does <u>NOT</u> clear the fence

6. Let's say that Bryce Harper tries to hit the ball again, but this time with an angle of 23° with the horizontal. Using the new parametric equations, does he hit a home run? Draw a picture to help you answer the question.

DEGREE MODE

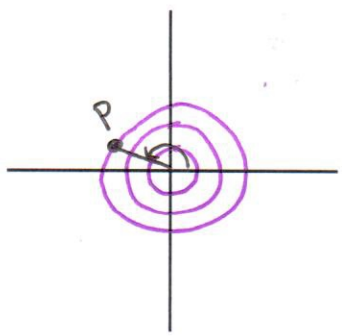
$$x=(100\cos 23)t \text{ and } y=-16t^2+(100\sin 23)t+3$$

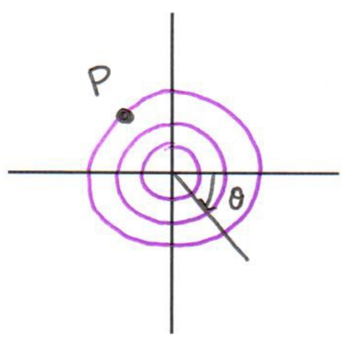


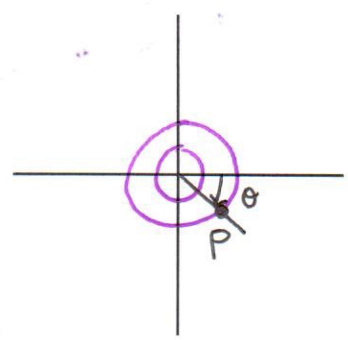
still no home run!

Plot the points with the given polar coordinates.

7. $\left(3,\frac{5\pi}{6}\right)$ **8.** $\left(-3,-\frac{2\pi}{6}\right)$ **9.** $(2,-45^\circ)$







Use algebra to find the rectangular coordinates of the points with the given polar coordinates. $x=r\cos\theta$ $y=r\sin\theta$

10. $\left(-1,-\frac{\pi}{3}\right)$ (r θ)

$x=-1\cos-\frac{\pi}{3}=-\frac{1}{2}$

$y=-1\sin-\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$\boxed{\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)}$

11. $\left(\sqrt{3},\frac{5\pi}{6}\right)$ (r θ)

$x=\sqrt{3}\cos\frac{5\pi}{6}=-\frac{3}{2}$

$y=\sqrt{3}\sin\frac{5\pi}{6}=\frac{\sqrt{3}}{2}$

$\boxed{\left(-\frac{3}{2},\frac{\sqrt{3}}{2}\right)}$

Use algebra to find the polar coordinates of the points with the given rectangular coordinates. $\theta=\tan^{-1}\left(\frac{y}{x}\right)$ $r=\pm\sqrt{x^2+y^2}$

12. $(2,-1)$ (x y)

$\theta=\tan^{-1}\left(\frac{-1}{2}\right)=-26.57^\circ$

$r=\pm\sqrt{2^2+(-1)^2}=\pm\sqrt{5}$

$\boxed{(\sqrt{5},-26.57^\circ)\ (-\sqrt{5},153.43)}$

13. $(1,3)$ (x y)

$\theta=\tan^{-1}\left(\frac{3}{1}\right)=71.57^\circ$

$r=\pm\sqrt{1^2+3^2}=\pm\sqrt{10}$

$\boxed{(\sqrt{10},71.57^\circ)\ (-\sqrt{10},251.57^\circ)}$

Convert the following equations from rectangular form to polar form.

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ $r^2 = x^2+y^2$ $x = r\cos\theta$ $y = r\sin\theta$

14. $\underbrace{x^2+y^2}_{r^2} - 8\underbrace{y}_{r\sin\theta} = 0$

$\boxed{r^2 - 8r\sin\theta = 0}$

15. $3\underbrace{x}_{r\cos\theta} - 6\underbrace{y}_{r\sin\theta} + 2 = 0$

$\boxed{3r\cos\theta - 6r\sin\theta + 2 = 0}$

Convert the following equations from polar form to rectangular form.

16. $r = 2\cos\theta$ Need an r

$r \cdot r = r \cdot 2\cos\theta$

$\underbrace{r^2}_{x^2+y^2} = 2\underbrace{r\cos\theta}_{x}$

$\boxed{x^2+y^2 = 2x}$

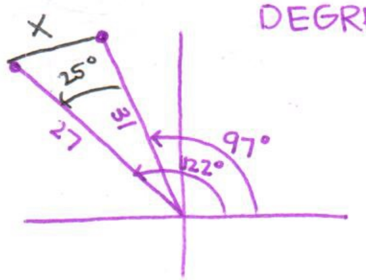
17. $r = 6\sec\theta$

$\cos\theta \; r = 6\left(\frac{1}{\cos\theta}\right) \cdot \cos\theta$

$r\cos\theta = 6$

$\boxed{x = 6}$

18. The locations of two UFOs in the night sky above BYU Stadium, given in polar coordinates, are (27*mi*, 122°) and (31*mi*, 97°). Find the distance between the two UFOs. First, draw a picture.



DEGREE MODE

$x^2 = 27^2 + 31^2 - 2(27)(31)\cos 25°$

$x^2 = 1690 - 1674\cos 25°$

$\sqrt{x^2} = \sqrt{172.84}$

$\boxed{x = 13.15 \text{ miles}}$

WITHOUT A CALCULATOR, determine the number of petals on each rose curve, given the following equations. Then list how long each petal is.

19. $r = 3\cos 6\theta$

$n=6$

$\boxed{12 \text{ petals}}$

$a=3$

$\boxed{3 \text{ units long}}$

20. $r = 3\sin 8\theta$

$n=8$

$\boxed{16 \text{ petals}}$

$a=3$

$\boxed{3 \text{ units long}}$

21. $r = 4\cos 17\theta$

$n=17$

$\boxed{17 \text{ petals}}$

$a=4$

$\boxed{4 \text{ units long}}$

WITHOUT A CALCULATOR, determine what type of limaçon is represented by the following equations.

22. $r = 4 + 6\cos\theta$

$a=4$ $b=6$ $\frac{4}{6} = \frac{2}{3} < 1$

$\boxed{\text{inner loop}}$

23. $r = 2 - \cos\theta$

$a=2$ $b=1$ $\frac{2}{1} \geq 2$

$\boxed{\text{convex}}$

24. $r = 3 - 3\sin\theta$

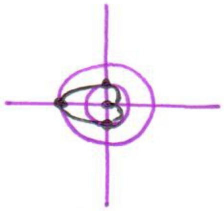
$a=3$ $b=3$ $\frac{3}{3} = 1$

$\boxed{\text{cardioid}}$

WITHOUT A CALCULATOR, Sketch the graph of each polar curve. Then name the type of polar curve.

25. $r = 1 - \cos\theta$

θ	r
0	0
$\pi/2$	1
π	2
$3\pi/2$	1
$\pi/4$	0.29



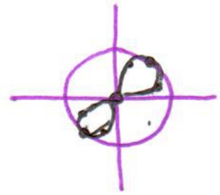
symm to x-axis
$a=1$ $b=1$ $\frac{a}{b}=1$

Limacon
(cardioid)

26. $r^2 = \sin 2\theta$

$r = \sqrt{\sin 2\theta}$

θ	r
0	0
$\pi/2$	0
π	0
$3\pi/2$	0
$\pi/4$	1
$\pi/6$	0.93
$\pi/3$	0.93



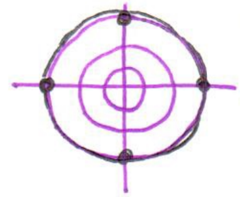
symm to origin

Lemniscate

27. $r = 3$

circle

θ	r
0	3
$\pi/2$	3
π	3
$3\pi/2$	3
$\pi/4$	3

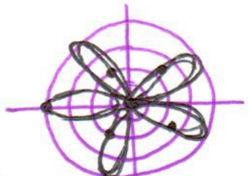


CIRCLE

28. $r = -4\cos 5\theta$

θ	r
$2\pi/3$	2

θ	r
0	-4
$\pi/2$	0
$3\pi/2$	0
π	4
$\pi/4$	2.83
$3\pi/4$	-2.83

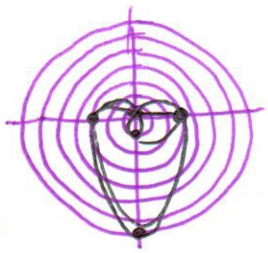


symm. to x-axis
5 petals
4 units long

ROSE CURVE

29. $r = 3 - 4\sin\theta$

θ	r
0	3
$\pi/2$	-1
π	3
$3\pi/2$	7
$\pi/4$	0.17

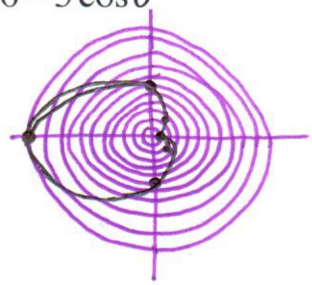


symm. to y-axis
$a=3$ $b=4$ $\frac{a}{b}<1$

LIMACON (inner loop)

30. $r = 6 - 5\cos\theta$

θ	r
0	1
$\pi/2$	6
π	11
$3\pi/2$	6
$\pi/4$	2.46

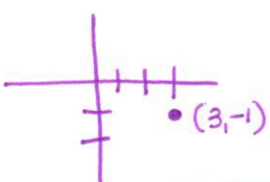


symm. to x-axis
$a=6$ $b=5$ $2>\frac{6}{5}>1$

LIMACON (dimpled)

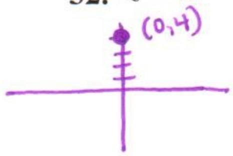
Graph each number in the complex plane, and find its absolute value.

31. $z = 3 - i$



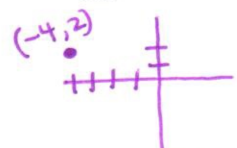
$|z| = \sqrt{(3)^2 + (-1)^2}$
$= \sqrt{9+1}$
$\boxed{|z| = \sqrt{10}}$

32. $z = 4i$ $\quad 0+4i$



$|z| = \sqrt{0^2 + 4^2}$
$= \sqrt{16}$
$\boxed{|z| = 4}$

33. $z = -4 + 2i$



$|z| = \sqrt{(-4)^2 + (2)^2}$
$= \sqrt{16+4}$
$\boxed{|z| = \sqrt{20}}$

Express each complex number in polar form.

34. $3 + \sqrt{2}i$ $\quad a>0$

$\theta = \tan^{-1}\left(\frac{\sqrt{2}}{3}\right) = 0.44$

$r = \sqrt{3^2 + (\sqrt{2})^2} = \sqrt{9+2} = \sqrt{11}$

$\boxed{\sqrt{11}\left(\cos 0.44 + i\sin 0.44\right)}$

35. $-5 + 8i$ $\quad a<0$

$\theta = \tan^{-1}\left(\frac{8}{-5}\right) + \pi = 2.13$

$r = \sqrt{(-5)^2 + (8)^2} = \sqrt{25+64} = \sqrt{89}$

$\boxed{\sqrt{89}\left(\cos 2.13 + i\sin 2.13\right)}$

Graph each complex number on a polar grid. Then express it in rectangular form.

36. $z = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$r = 3 \quad \theta = \frac{\pi}{2}$

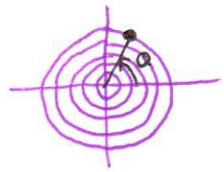
$3\left[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right]$

$= 3\left[0 + 1i\right]$

$= \boxed{0 + 3i}$

37. $z = 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$r = 5 \quad \theta = \frac{\pi}{3}$



$5\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$

$= 5\left[\frac{1}{2} + \frac{\sqrt{3}}{2}i\right]$

$= \boxed{\frac{5}{2} + \frac{5\sqrt{3}}{2}i}$

Find each product or quotient. Then express it in rectangular form.

38. $-2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) \bullet -4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= (-2)(-4)\left[\cos\left(\frac{5\pi}{6} + \frac{\pi}{3}\right) + i\sin\left(\frac{5\pi}{6} + \frac{\pi}{3}\right)\right]$

$= 8\left[\cos\left(\frac{5\pi}{6} + \frac{2\pi}{6}\right) + i\sin\left(\frac{5\pi}{6} + \frac{2\pi}{6}\right)\right]$

$= 8\left[\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right]$ ✓

$8\left(-\frac{\sqrt{3}}{2} + -\frac{1}{2}i\right)$

$\boxed{-4\sqrt{3} - 4i}$

39. $6\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) \div 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$= \frac{6}{3}\left[\cos\left(\frac{\pi}{4} - \frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{4} - \frac{\pi}{2}\right)\right]$

$= 2\left[\cos\left(\frac{\pi}{4} - \frac{2\pi}{4}\right) + i\sin\left(\frac{\pi}{4} - \frac{2\pi}{4}\right)\right]$

$= 2\left[\cos -\frac{\pi}{4} + i\sin -\frac{\pi}{4}\right]$ ✓

$2\left(\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2}i\right)$

$\boxed{\sqrt{2} - \sqrt{2}i}$

Find each power. Then express it in rectangular form.

40. $(4 - i)^5$

$a > 0 \quad \theta = \tan^{-1}\left(-\frac{1}{4}\right) = -0.25$

$r = \sqrt{4^2 + (-1)^2} = \sqrt{17}$

$= \left[\sqrt{17}\left(\cos(-0.25) + i\sin(-0.25)\right)\right]^5$

$= (\sqrt{17})^5\left(\cos 5(-0.25) + i\sin 5(-0.25)\right)$ ✓

$= 1191.58\left(0.32 + -0.95i\right)$

$= \boxed{381.31 - 1132.0i}$

41. $(\sqrt{2} + 3i)^4$

$a > 0 \quad \theta = \tan^{-1}\left(\frac{3}{\sqrt{2}}\right) = 1.13$

$r = \sqrt{(\sqrt{2})^2 + (3)^2} = \sqrt{11}$

$\left[\sqrt{11}\left(\cos 1.13 + i\sin 1.13\right)\right]^4$

$= (\sqrt{11})^4\left(\cos 4(1.13) + i\sin 4(1.13)\right)$ ✓

$= 121\left(-0.19 + -0.98i\right)$

$= \boxed{-22.99 + -118.58i}$

Find all the distinct *p*th roots of the complex number.

42. cube roots of $6 - 4i$

$\theta = \tan^{-1}\left(-\frac{4}{6}\right) = -0.59 \quad r = \sqrt{6^2 + (-4)^2} = \sqrt{52} = 7.21$

$p = 3 \quad n = 0, 1, 2 \quad a > 0$

$p = 3,\ n = 0$

$= 7.21^{1/3}\left[\cos\left(\frac{-0.59 + 2(0)\pi}{3}\right) + i\sin\left(\frac{-0.59 + 2(0)\pi}{3}\right)\right] = \boxed{1.89 - 0.38i}$

$p = 3 \quad n = 1$

$= 7.21^{1/3}\left[\cos\left(\frac{-0.59 + 2(1)\pi}{3}\right) + i\sin\left(\frac{-0.59 + 2(1)\pi}{3}\right)\right] = \boxed{-0.62 + 1.83i}$